

Mathematical Validation of Trend Functions Under Deterministic Linear Growth: A Comparative Study

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Abstract

This study examines Fidelity Bank asset values across three trend observation periods. Firstly, mathematical formulations of the problem were developed to establish the share-price rate of change for three different trend functions, namely Linear, Quadratic and Cubic, each of which has a distinct impact on the modelled share prices of Fidelity Bank. We analyze column-wise trends, and evaluate row-wise descriptive statistics and trend functions using error metrics. Results show that all assets follow a perfect linear trend where the second and third observations are exactly twice and thrice the first. Mean and standard deviation increase proportionally across assets, with constant relative risk. Also, a two-way ANOVA based on Snedecor's F-distribution is applied. Results reveal statistically significant differences in both trend-function and asset effects, with perfect linear scaling. The linear trend function fits perfectly with zero error. In addition, Quantile-Quantile (Q-Q) plots were used to compare the three trend-generated price series, confirming that they are exactly proportional by construction. The proposed framework offers a useful methodological benchmark for comparing trend-fitting approaches prior to their application to empirical, stochastically varying share-price data.

Keywords: Share price, Trend Functions, Market, Asset and Valuation.

1 Introduction

Investment decisions in financial markets fundamentally depend on the ability to model and forecast asset behavior over time. Trend functions provide a mathematical representation of how asset values evolve, while Analysis of Variance (ANOVA) quantifies the directional movements. The validity of these models is therefore critical for selecting appropriate strategies in portfolio management and risk assessment. However, the valuation and modeling of bank assets are central to financial decision-making, risk assessment, and investment planning. Accurate identification of growth patterns and trend functions enables institutions to forecast performance, allocate resources efficiently, and manage exposure to volatility. In practice, asset values are expected to exhibit stochastic behavior due to market forces, regulatory changes, and macroeconomic conditions. More so, when data show consistent and proportional growth, it

becomes necessary to evaluate whether simple deterministic models are sufficient, or whether more complex series and probabilistic approaches are required.

A substantial body of literature has examined stock market prices using diverse methods. For instance, [1] applied matrix algebra to model market trend functions. [2] examined the share prices of Access Bank and Fidelity Bank using a stochastic model, presenting predicted share prices for the merged bank. In a related study, [3] applied matrix methods to Dangote stock market prices, illustrating investment outcomes in different forms. Similarly, [4] examined matrix differential calculus with a stochastic term and measurable spaces for investment planning. In another study, [5] examined a stochastic system with changing measures of wealth for each corporate investor through linear and quadratic returns. Furthermore, [6] investigated a system of stochastic differential equations with emphasis on variations in drift parameters. [7] carried out a stochastic analysis of two asset values. [8] examined the stability and controllability of the stock exchange market by developing a vector-valued stochastic differential system with control. [9] examined the application of various stochastic volatility models in determining optimal investment strategies. Subsequently, [10] studied a stochastic model of stock market price fluctuation. Several other scholars have likewise addressed the modelling of stock prices in different forms [11–16].

This study is therefore motivated by the need to examine three distinct trend functions using error metrics as a criterion for selection. This paper contributes by demonstrating how descriptive statistics, trend analysis, and ANOVA can be applied to a small-scale asset dataset.

The paper is set out as follows: Section 2 presents materials and methods, Section 3 presents results and discussion, while the paper is concluded in Section 4.

2 Materials and Methods

In this section, we present three foundational results that underpin the trend functions used in this study, following [1]. The following theorems are stated and proved:

Theorem 2.1 (Linear Growth). Let $S(t) = S_0 + mt$, where $S_0, m \in \mathbb{R}$ and $t \geq 0$. Then $\frac{dS}{dt} = m$.

Proof. Using linearity of differentiation and the power rule $\frac{d}{dt}t^n = nt^{n-1}$,

$$\frac{dS}{dt} = \frac{d}{dt}S_0 + \frac{d}{dt}(mt) = 0 + m \cdot 1 = m,$$

since S_0 is a constant. ■

Theorem 2.2 (Quadratic Growth). Let $S(t) = S_0 + at + bt^2$, where $S_0, a, b \in \mathbb{R}$ and $t \geq 0$. Then $\frac{dS}{dt} = a + 2bt$.

Proof. Applying linearity and the power rule termwise:

$$\frac{dS}{dt} = \frac{d}{dt}S_0 + \frac{d}{dt}(at) + \frac{d}{dt}(bt^2) = 0 + a \cdot 1 + b \cdot 2t^{2-1} = a + 2bt,$$

since S_0 is a constant. ■

Theorem 2.3 (Cubic Growth). Let $S(t) = S_0 + at + bt^2 + ct^3$, where $S_0, a, b, c \in \mathbb{R}$ and $t \geq 0$. Then $\frac{dS}{dt} = a + 2bt + 3ct^2$.

Proof. By linearity and the power rule,

$$\frac{dS}{dt} = 0 + a \cdot 1 + b \cdot 2t + c \cdot 3t^2 = a + 2bt + 3ct^2,$$

since S_0 is a constant. ■

2.1 Matrix Algebra

This is a set of rules for adding, multiplying, and manipulating matrices the same way we do with numbers, but with special rules. For example:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad (1)$$

That is a 2×3 matrix: 2 rows, 3 columns.

2.2 Rate of Change of Asset over Time

This is the derivative of price with respect to time; it tells us how fast the stock is moving at a given instant. For an asset with price $S(t)$ at time t :

$$\text{Rate of change} = \frac{dS}{dt} = \lim_{\Delta t \rightarrow 0} \frac{S(t + \Delta t) - S(t)}{\Delta t}.$$

2.2.1 Problem Formulation

Here we explore the application of matrix algebra to model the share prices of Fidelity Bank, considering three different trends: Linear, Quadratic and Cubic. Let $K_1(t)$, $K_2(t)$ and $K_3(t)$ represent the share prices under the three trend functions at time t :

$$K_1(t) = \begin{pmatrix} \mu_1 S'_{011} t & \mu_1 S'_{012} t & \mu_1 S'_{013} t \\ \mu_1 S'_{021} t & \mu_1 S'_{022} t & \mu_1 S'_{023} t \\ \mu_1 S'_{031} t & \mu_1 S'_{032} t & \mu_1 S'_{033} t \end{pmatrix} \quad (2)$$

$$K_2(t) = \begin{pmatrix} \mu_2 S''_{011} t^2 & \mu_2 S''_{012} t^2 & \mu_2 S''_{013} t^2 \\ \mu_2 S''_{021} t^2 & \mu_2 S''_{022} t^2 & \mu_2 S''_{023} t^2 \\ \mu_2 S''_{031} t^2 & \mu_2 S''_{032} t^2 & \mu_2 S''_{033} t^2 \end{pmatrix} \quad (3)$$

$$K_3(t) = \begin{pmatrix} \mu_3 S'''_{011} t^3 & \mu_3 S'''_{012} t^3 & \mu_3 S'''_{013} t^3 \\ \mu_3 S'''_{021} t^3 & \mu_3 S'''_{022} t^3 & \mu_3 S'''_{023} t^3 \\ \mu_3 S'''_{031} t^3 & \mu_3 S'''_{032} t^3 & \mu_3 S'''_{033} t^3 \end{pmatrix} \quad (4)$$

where $K_1(t)$, $K_2(t)$ and $K_3(t)$ denote the share price of Fidelity Bank at time t under the linear, quadratic, and cubic trend functions respectively; $S'_{t011} + \dots + S'_{t044}$, $S''_{t011} + \dots + S''_{t044}$ and $S'''_{t011} + \dots + S'''_{t044}$ represent the monthly initial share price of Fidelity Bank at time t under the respective trend functions; and μ_1 , μ_2 and μ_3 denote the expected instantaneous growth rate of the stock.

The share-price values used in Table 1 were constructed to satisfy exact proportional scaling across the three trend functions, so as to provide an unambiguous, controlled comparison of the fitting performance of the linear, quadratic and cubic trend functions. Real share prices observed in efficient capital markets are, by contrast, generally expected to exhibit stochastic rather than perfectly deterministic behaviour [19]; the deterministic dataset analysed here should therefore be interpreted as an illustrative benchmark for the trend-fitting methodology rather than as an empirical forecast of actual Fidelity Bank market prices.

2.2.2 Method of Analysis

Here, we apply the rate of change in solving Equations (2)–(4) according to their respective trend functions. The rate of change of an investment refers to how the value of the investment changes over time.

$$\frac{dK_1(t)}{dt} = \begin{pmatrix} \mu_1 S'_{011} & \mu_1 S'_{012} & \mu_1 S'_{013} \\ \mu_1 S'_{021} & \mu_1 S'_{022} & \mu_1 S'_{023} \\ \mu_1 S'_{031} & \mu_1 S'_{032} & \mu_1 S'_{033} \end{pmatrix} \quad (5)$$

$$\frac{dK_2(t)}{dt} = \begin{pmatrix} 2\mu_2 S''_{011}t & 2\mu_2 S''_{012}t & 2\mu_2 S''_{013}t \\ 2\mu_2 S''_{021}t & 2\mu_2 S''_{022}t & 2\mu_2 S''_{023}t \\ 2\mu_2 S''_{031}t & 2\mu_2 S''_{032}t & 2\mu_2 S''_{033}t \end{pmatrix} \quad (6)$$

$$\frac{dK_3(t)}{dt} = \begin{pmatrix} 3\mu_3 S'''_{011}t^2 & 3\mu_3 S'''_{012}t^2 & 3\mu_3 S'''_{013}t^2 \\ 3\mu_3 S'''_{021}t^2 & 3\mu_3 S'''_{022}t^2 & 3\mu_3 S'''_{023}t^2 \\ 3\mu_3 S'''_{031}t^2 & 3\mu_3 S'''_{032}t^2 & 3\mu_3 S'''_{033}t^2 \end{pmatrix} \quad (7)$$

2.3 Validation via Error Metrics

To confirm optimality, we compute:

1. Mean Absolute Error

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (8)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (9)$$

where y_i is the actual share price of Fidelity Bank, $\hat{y}_i$ is the corresponding predicted value, n is the number of data points, and Σ is the summation symbol, indicating the sum of the values.

2.4 Snedecor's F-Distribution

Assuming there are N stocks in the capital market, let $S_i, i = 1, 2, \dots, N$ be the initial stock prices for T trading days. Considering three finite matrices of sizes n_1 and n_2 , via a 3×3 matrix, whose variances are σ_1^2 and σ_2^2 respectively.

Now let

$$S_1^2 = \frac{1}{n} \sum_{i=1}^n (X_{1i} - \bar{X})^2 \quad \text{and} \quad S_2^2 = \frac{1}{n} \sum_{i=1}^n (X_{2i} - \bar{X})^2$$

represent the variances of the first and second predicted values. For simplicity, the F-distribution is defined as the ratio of χ^2 variates, which gives:

$$F = \frac{x_{n_1}^2/n_1}{x_{n_2}^2/n_2} = \frac{(n_1 - 1)S_1^2/\sigma_1^2}{n_1} \bigg/ \frac{(n_2 - 1)S_2^2/\sigma_2^2}{n_2} = \frac{S_1^2}{\sigma_1^2} \bigg/ \frac{S_2^2}{\sigma_2^2} \quad (10)$$

3 Results and Discussion

To demonstrate the closing share market price performance of Fidelity Bank, the following parameter values were used: $\mu_1 = \mu_2 = \mu_3 = 0.8$, $t = 1.00$.

Table 1 reports the nine entries of the 3×3 share-price matrices $K_1(t)$, $K_2(t)$ and $K_3(t)$ defined in Equations (2)–(4); each row therefore corresponds to one matrix entry, indexed as Asset 1 through Asset 9 in the two-way ANOVA of this section (Table 2) and in the post-hoc comparisons of Table 3, while the three columns correspond to the linear, quadratic and cubic trend functions.

Table 1: Asset values according to their respective trends with investment mean and standard deviation

Row (Asset)	Linear Trend $K_1(t)$	Quadratic Trend $K_2(t)$	Cubic Trend $K_3(t)$	Average	STD Dev
1	332	664	996	664.00	332.00
2	49.6	99.2	148.8	99.20	49.60
3	110.4	220.8	331.2	220.80	110.40
4	48.8	97.6	146.4	97.60	48.80
5	96.8	193.6	290.4	193.60	96.80
6	64.8	129.6	194.4	129.60	64.80
7	111.2	222.4	333.6	222.40	111.20
8	64	128	192	128.00	64.00
9	307.2	614.4	921.6	614.40	307.20

All three observations show perfect proportional scaling. The column means increase from $131.76 \rightarrow 263.51 \rightarrow 395.27$. There is no volatility across assets, since each asset's standard deviation and mean scale together as a direct consequence of the perfect linear growth by which the dataset was constructed. Asset 1 has the highest mean (664) and highest dispersion (332); Asset 4 has the lowest mean (97.6) and lowest dispersion (48.8), see Table 1.

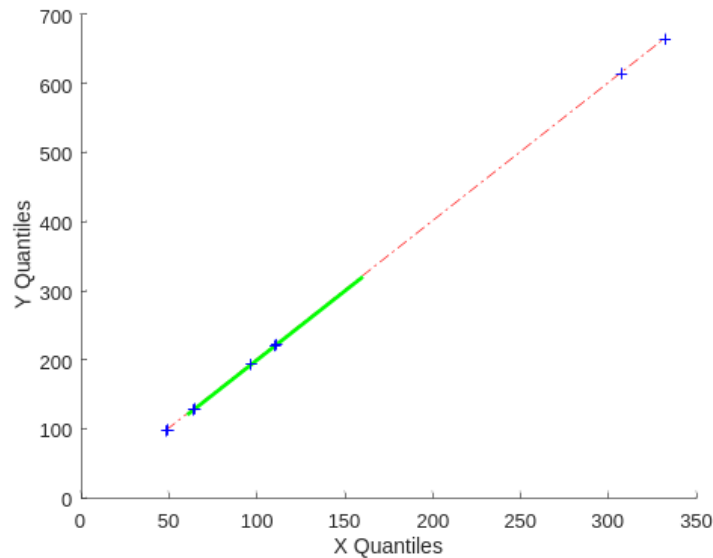


Figure 1: Statistical Quantile-Quantile (QQ) plots of Assets: Linear, Quadratic Trends

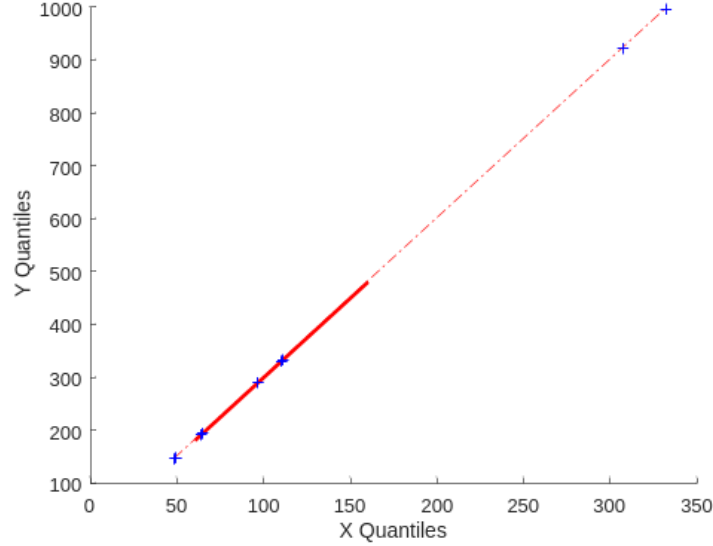


Figure 2: Statistical Quantile-Quantile (QQ) plots of Assets: Linear, Cubic Trends

Figures 1–3 present quantile-quantile (QQ) plots comparing each pair of trend-generated series (Linear–Quadratic, Linear–Cubic, and Quadratic–Cubic). In each case, the plotted quantiles fall exactly on a straight reference line, which is the expected outcome given that, by construction, the Quadratic and Cubic series are exact scalar multiples of the Linear series (Table 1). The slope of each line reflects the constant multiplicative factor between the two series being compared, while the near-zero intercept confirms the absence of an additive offset between them. A QQ-plot-based assessment of distributional shape [17] would ordinarily be applied to independently sampled empirical data; here, the plots instead illustrate the internal consistency of the deterministic construction of the dataset rather than constituting a formal test of the normality of observed market returns. This distinction should be kept in mind when generalising the result to real, stochastically varying share-price data.

Table 2: Two-Way ANOVA

Source of variation	SS	df	MS	F-cal.	F-crit. ($\alpha = 0.05$)	p-value	Decision
Between trend functions (Linear/Quadratic/Cubic)	314,580.59	2	157,290.30	1200.69	3.63	< 0.0001	Reject H_0
Between matrix components (Assets 1–9)	312,580.46	8	39,072.56	298.13	2.59	< 0.0001	Reject H_0
Residual/Error	2,098.13	16	131.13				
Total	316,678.72	26					

Hypothesis Tests

Test A: Trend-Function Effect

$$H_0 : \text{Asset-1} = \text{Asset-2} = \text{Asset-3} \quad H_1 : \text{At least one trend mean differs}$$

$$\text{Result: } F_{2,16} = 1200.69 > 3.63 \Rightarrow \text{Reject } H_0$$

The growth from 131.76 $\rightarrow$ 263.51 $\rightarrow$ 395.27 is statistically significant. The trend-function factor has a significant effect on the modelled share-price level.

Test B: Matrix-Component (Asset) Effect

$$H_0 : \text{Asset-1} = \text{Asset-2} = \dots = \text{Asset-9} \quad H_1 : \text{At least one asset mean differs}$$

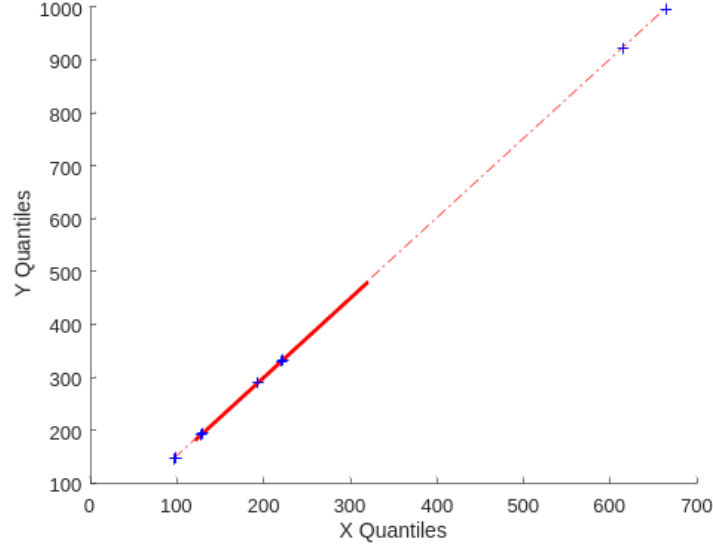


Figure 3: Statistical Quantile-Quantile (QQ) plots of Assets: Quadratic, Cubic Trends

$$\text{Result: } F_{8,16} = 298.13 > 2.59 \Rightarrow \text{Reject } H_0$$

The 9 assets do not have the same average level. Asset 1 (664.00) and Asset 9 (614.40) are significantly larger than Asset 4 (97.60).

Table 3: Comparison of Means (Critical Difference = 18.72)

Asset Pair	Mean Difference	Significant
Asset 1 versus Asset 4	566.40	Yes
Asset 9 versus Asset 4	516.80	Yes
Asset 1 versus Asset 2	564.80	Yes
Asset 7 versus Asset 4	124.80	Yes

All matrix components (assets) with large trend values are significantly different from those with small trend values, based on the critical-difference post-hoc procedure [18].

Effect Size

$$\eta_{\text{TIME}}^2 = \frac{SS_{\text{TIME}}}{SS_{\text{TOTAL}}} = \frac{314580.59}{316678.72} = 0.9934, \quad \eta_{\text{ASSET}}^2 = \frac{SS_{\text{ASSET}}}{SS_{\text{TOTAL}}} = \frac{312580.46}{316678.72} = 0.9871 \quad (11)$$

99.3% of total variation in Table 2 is explained by the trend-function factor, and 98.7% by the matrix-component (asset) factor; these very high effect sizes reflect the deterministic, exactly proportional construction of the dataset described in Section 2.3.1, rather than the explanatory power typically observed in empirical market data.

More generally, the ANOVA results indicate that matrix components (Assets) 1 and 9 belong to a statistically different class from Asset 4. However, because the dataset was constructed under a single deterministic growth direction, the ANOVA cannot speak to differences in risk (variance) that would be expected to arise under stochastic growth conditions.

4 Conclusion

The illustrative Fidelity Bank asset data analysed in this study demonstrate uniform proportional growth across all matrix components and trend functions. Based on the computed error metrics, the linear trend function provides the best fit to this deterministic dataset. Average and standard deviation scale linearly with asset size, and the two-way ANOVA confirms that both the trend-function and matrix-component (asset) effects are statistically significant, consistent with the deterministic linear growth pattern built into the data. Assets 1 and 9 exhibit the highest mean levels, while relative risk, measured by the coefficient of variation, remains constant at 0.5 across all assets. The QQ plots confirm that the three trend-generated series are related by an exact linear (proportional) transformation, which is useful as a methodological benchmark rather than as evidence of empirical risk equivalence. Because real financial markets typically exhibit stochastic rather than purely deterministic price behaviour [19], future research should extend this framework to longer, empirically observed time series with genuine price fluctuations, in order to assess the impact of volatility regimes on trend-function selection for stock-market decision-making.

References

- [1] Wokoma, D. A., & Amadi, I. U. (2026). Analysis of market trend models and asset valuation under inflationary pressure. *International Journal of Innovative Mathematics, Statistics and Energy Policies*, 14(1), 231–241.
- [2] Osu, B. O., Emenyonu, S. C., Ogbogbo, C. P., & Olunkwa, C. (2024). Markov models on share price movements in Nigeria stock market capitalization. *Journal of Statistics Applications & Probability*, 13(1). <https://doi.org/10.18576/jsap/130124>
- [3] Amadi, I. U., Onyegbuchulem, C., & Odu-Ndom, N. K. (2025). Effect of different trend functions with stochastic terms and measurable spaces for capital markets. *Asian Journal of Pure and Applied Mathematics*, 8(2), 234–248.
- [4] Amadi, I. U., Osu, B. O., Nwobi, F. N., & Anthony, C. (2025). A matrix approach in predicting stock market prices with regularly varying perturbations. *Asian Journal of Pure and Applied Mathematics*, 7(1), 386–399.
- [5] Amadi, I. U., Ogbogbo, C. P. N., Davies, I., & Katsekpor, T. (2024). A stochastic model for the variation of Fourier series expansion with time delay arising in financial market price changes. *International Journal of Mathematical Sciences and Optimization Theory and Applications*, 10(3), 140–153.
- [6] Wokoma, D. S. A. (Bishop), Amadi, I. U., & Aboko, I. S. (2022). A stochastic model for stock market price variation. *Asian Research Journal of Mathematics*, 18(4), 41–49.
- [7] Osu, B. A., & Amadi, I. U. (2022). A stochastic analysis of stock market price fluctuations for capital market. *Journal of Applied Mathematics and Computation*, 6(1), 85–95.
- [8] Davies, I., Amadi, I. U., Amadi, C. P., Ateke, R. C., & Oworndah, N. S. (2023). Stability and controllability analysis of stochastic model for stock market prices. *International Journal of Statistics and Applied Mathematics*, 8(4), 55–62.

- [9] Azor, P. A., & Amadi, U. C. (2024). Mathematical modeling of an investor's wealth with different stochastic volatility models. *Communication in Physical Sciences*, 11(2), 355–372.
- [10] Osu, B. O. (2010). A stochastic model of the variation of the capital market price. *International Journal of Trade, Economics and Finance*, 1(3), 297–302.
- [11] Gupta, B. D. (2010). *Mathematical physics* (4th ed.). Vikas Publishing House.
- [12] Nwobi, F. N., & Azor, P. A. (2022). An empirical approach for the variation in capital market price changes. *International Journal of Statistics and Mathematics*, 8(1), 164–172.
- [13] Osu, B. O., Okoroafor, A. C., & Olunkwe, C. (2009). Stability analysis of stochastic model of stock market price. *African Journal of Mathematics and Computer Science Research*, 2(6), 98–102.
- [14] Osu, B. O., Amadi, I. U., & Azor, P. A. (2022). Analytical solution of time-varying investment returns with random parameters. *Sebha University Journal of Pure and Applied Sciences*, 21(2). <https://doi.org/10.51984/JOPAS.v2112.1857>
- [15] Adeosun, M. E., Edeki, S. O., & Ugbebor, O. O. (2015). Stochastic analysis of stock market price models: A case study of Nigerian Stock Exchange (NSE). *WSEAS Transactions on Mathematics*, 14, 353–363.
- [16] Luenberger, D. G. (2013). *Investment science*. Oxford University Press.
- [17] Shapiro, S. S., & Wilk, M. B. (1965). An analysis of variance test for normality (complete samples). *Biometrika*, 52(3–4), 591–611. <https://doi.org/10.1093/biomet/52.3-4.591>
- [18] Montgomery, D. C. (2019). *Design and analysis of experiments* (10th ed.). Wiley.
- [19] Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2), 383–417.