ROGUE WAVE DYNAMICS IN INSECURITY PROPAGATION IN SOUTH-EAST NIGERIA: A MACHINE-LEARNING ASSISTED NONLINEAR OPTIMAL CONTROL FRAMEWORK FOR NON-KINETIC INTERVENTION

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Abstract

Insecurity in South-East Nigeria is **characterised** by sudden, extreme surges in violent activity that defy explanation by classical linear or epidemic-type models. This study proposes a nonlinear rogue-wave framework that captures these abrupt spikes through modulation instability arising from nonlinear interactions among armed groups, socio-economic stressors, and state responses. Using security incident data from the Armed Conflict Location and Event Data Project (ACLED) and the Nigeria Security Tracker (2015–2025), machine-learning techniques, including Gaussian Process Regression, Support Vector Regression, Long Short-Term Memory networks, and reinforcement learning, are integrated for parameter estimation, early detection, and adaptive control tuning. An optimal non-kinetic control problem is formulated with three time-dependent controls—intelligence-led operations, community engagement, and socio-economic interventions—with optimality conditions derived via Pontryagin's Maximum Principle. A fractional-order extension incorporating memory effects is developed, yielding $q = 0.85$ as the optimal fit to ACLED data (Root Mean Square Error 0.042). Numerical simulations demonstrate that the proposed strategies suppress rogue-wave insecurity spikes by 73.4% in theoretical settings, with expected real-world effectiveness of 41–52% given implementation constraints. The **machine-learning early-warning system achieves 89.2% accuracy, with an Area Under the Curve of 0.94**. Sensitivity analysis confirms the fractional-order parameter's critical role in modulating system dynamics. This mathematically rigorous, policy-relevant framework offers actionable insights for mitigating extreme insecurity outbreaks in South-East Nigeria through timely, data-driven non-kinetic interventions.

Keywords: Rogue waves; Insecurity dynamics; Machine learning; Nonlinear optimal control; Non-kinetic intervention; South-East Nigeria; Fractional calculus

2010 Mathematics Subject Classification: 93C10; 93C95; 34A08; 49J15; 68T05

1 Introduction

Insecurity remains a persistent socio-political and economic challenge in many regions of the world, particularly in fragile and post-conflict environments. In South-East Nigeria, insecurity has evolved beyond sporadic violent incidents into a complex, nonlinear phenomenon **characterised** by sudden surges in attacks, disruptions of socio-economic activities, and erosion of public trust in governance. These abrupt escalations—often unpredictable in timing and magnitude—pose significant challenges to traditional security planning and response strategies (Onuoha, 2018).

Classical mathematical models of insecurity and conflict dynamics have largely relied on linear, compartmental, or epidemic-type frameworks, which assume smooth evolution and gradual changes in attack intensity (Hethcote, 2000; Murray, 2002). While such models provide useful baseline insights, they fail to adequately capture extreme, short-lived, high-impact events, such as coordinated attacks or sudden outbreaks of violence, frequently observed in South-East Nigeria. Recent studies have **emphasised** the need for nonlinear and instability-driven modelling approaches capable of describing these sharp transitions and explosive dynamics (Ejinkonye & Abdullahi, 2025; Ejinkonye & Mankilik, 2025).

In nonlinear physics and applied mathematics, rogue waves are rare, high-amplitude events arising from nonlinear interactions and modulation instability in complex systems (Akhmediev et al., 2009; Zakharov & Ostrovsky, 2009). Originally studied in oceanography, rogue-wave theory has been successfully extended to optics, plasma physics, finance, and traffic flow to explain extreme events that emerge unexpectedly from otherwise moderate dynamics. This conceptual framework provides a powerful and novel lens for understanding sudden spikes in insecurity, where **localised** nonlinear interactions among armed groups, socio-economic stressors, and institutional responses generate disproportionately large impacts.

Beyond modelling, effective mitigation of insecurity increasingly requires non-kinetic intervention strategies, including intelligence-led policing, community engagement, economic empowerment, and information-driven governance. Unlike kinetic approaches, non-kinetic strategies aim to reduce violence by addressing its underlying drivers rather than relying solely on force. Mathematical optimal control theory offers a rigorous framework for designing such interventions by balancing effectiveness against economic and social costs (Lenhart & Workman, 2007). Previous applications of optimal control to conflict and crime dynamics have demonstrated its value in identifying cost-effective and sustainable policy responses (Ejinkonye et al., 2025; Ejinkonye & Omokoh, 2025).

However, a major limitation in existing models is the reliance on fixed or poorly estimated parameters, which reduces predictive accuracy in real-world settings. The growing availability of security-related data has made machine learning an essential tool for parameter estimation, pattern recognition, and early-warning detection of extreme events (Bishop, 2006; Varian, 2019). Integrating machine learning with nonlinear dynamical systems enables adaptive, data-driven control strategies that respond to evolving insecurity patterns and anticipate rogue-wave-like outbreaks before they fully develop (Ejinkonye & Oghenetega, 2025).

Furthermore, the persistence of socio-political vulnerabilities and historical grievances in conflict zones suggests that memory effects play a crucial role in insecurity propagation. Fractional calculus provides a natural framework for incorporating such memory effects, capturing the long-term influence of past events on current dynamics (Diethelm, 2010; Podlubny, 1999). The integration of fractional-order modelling with optimal control theory offers **an enhanced capability** for designing interventions that account for the lasting impact of socio-economic and political factors. Similar approaches have been successfully applied to disease outbreak

modelling in Nigeria (Ayansiji & Ejinkonye, 2025).

Recent deterministic modelling of intra-communal violence in Nigeria has represented violence dynamics through ordinary differential equations and examined stability and sensitivity, supporting the use of dynamical-systems approaches for insecurity analysis (Marcus et al., 2024).

Related work in South-South Nigeria has also used compartmental ordinary differential equation models to examine interconnected socio-economic and crime-related dynamics under intervention scenarios, illustrating the policy-oriented use of mathematical systems for complex social problems (Ohwojeheri et al., 2025).

In parallel, anomaly-detection models that integrate ACLED and GDELT data have been investigated for early warning of socio-political unrest, with emphasis on identifying sudden departures from prevailing behavioural patterns (Macis et al., 2024).

Machine-learning conflict forecasts based on ACLED-informed inputs have also been coupled with agent-based simulations, demonstrating the feasibility of data-driven conflict prediction at relatively fine spatial and temporal scales (Xue et al., 2025).

However, recent comparative evidence indicates that greater model complexity does not necessarily improve political-violence forecasts, reinforcing the need for careful out-of-sample validation and parsimonious predictive design (Chadefaux & Schincariol, 2025).

This paper makes several distinct contributions that extend our previous work. While Ejinkonye and Abdullahi (2025) introduced nonlinear modelling for insecurity, the present study **formalises** the rogue-wave analogy with rigorous mathematical conditions for modulation instability in the insecurity context. Unlike Ejinkonye and Oghenetega (2025), which proposed machine learning conceptually, this work implements Gaussian Process Regression, Support Vector Regression, Long Short-Term Memory networks, and Proximal Policy **Optimisation** algorithms with concrete performance metrics validated against ACLED and Nigeria Security Tracker data. We extend beyond integer-order models by incorporating fractional calculus with explicit justification for fractional-order values and comprehensive sensitivity analysis. Finally, this work provides the first complete Pontryagin-based optimal control **characterisation** for insecurity dynamics with three distinct non-kinetic interventions.

Objective of the Study

Motivated by these gaps, this study develops a machine-learning-assisted nonlinear optimal control framework to investigate rogue-wave dynamics in the propagation of insecurity in South-East Nigeria. The proposed model captures sudden insecurity spikes through nonlinear instability mechanisms, employs machine learning to enhance parameter estimation and prediction, incorporates optimal non-kinetic controls to suppress extreme events efficiently, and extends the analysis to fractional-order systems to account for memory effects. By combining nonlinear dynamics, artificial intelligence, fractional calculus, and control theory, this work provides both a mathematically rigorous and policy-relevant framework for understanding and mitigating insecurity in the region.

2 Review of Related Literature

2.1 Mathematical Modelling of Insecurity and Conflict

The application of mathematical models to understand conflict and insecurity dynamics has gained significant traction in recent decades. Early approaches drew heavily from epidemiological frameworks, treating insecurity as a contagious phenomenon that spreads through susceptible populations (Hethcote, 2000). The compartmental Susceptible-Infected-Recovered (SIR) model and its variants have been adapted to describe the diffusion of violent ideologies, recruitment into armed groups, and the spatial spread of conflict (Murray, 2002). While these models provide valuable insights into the aggregate dynamics of insecurity, they inherently assume smooth, gradual transitions between compartments, limiting their capacity to capture abrupt, explosive increases in violence.

Onuoha (2018) provided a comprehensive political economy analysis of insecurity in Nigeria, highlighting the complex interplay of economic marginalisation, political exclusion, and ethnic grievances. **The study emphasised** that insecurity in South-East Nigeria is not merely a security problem but a manifestation of deeper structural issues. This perspective necessitates modelling approaches that can capture the nonlinear feedback mechanisms through which socio-economic factors amplify or dampen conflict intensity.

The SEIR-type compartmental framework has been successfully applied to disease outbreak modelling in Nigeria (Ayansiji & Ejinkonye, 2025), demonstrating the utility of epidemic models for understanding propagation dynamics. However, insecurity dynamics differ fundamentally from disease transmission in that they involve intentional actors, strategic decision-making, and complex feedback loops that necessitate nonlinear modelling approaches.

2.2 Rogue Wave Theory and Extreme Event Modelling

Rogue wave theory emerged from oceanography to explain the sudden appearance of massive waves in otherwise moderate sea states (Zakharov & Ostrovsky, 2009). The mathematical foundation of rogue waves lies in the nonlinear Schrödinger equation, which describes modulation instability—a condition under which small perturbations grow exponentially, leading to **localised** high-amplitude events (Akhmediev et al., 2009). This framework has been successfully extended to various disciplines, including optics, plasma physics, and finance, where extreme events exhibit similar characteristics of sudden onset, high amplitude, and rapid decay.

The application of rogue wave theory to social and political systems is nascent but promising. Ejinkonye and Abdullahi (2025) first proposed the analogy between sudden violence outbreaks and rogue waves, arguing that nonlinear interactions among armed groups, state responses, and socio-economic stressors create conditions analogous to modulation instability. Ejinkonye and Mankilik (2025) extended this framework to herder-farmer conflicts in Delta State, Nigeria, demonstrating the utility of rogue wave models in capturing **localised** conflict spikes. These studies provide the foundation for the present work, which **formalises** the rogue wave analogy with rigorous mathematical conditions and empirical validation.

2.3 Machine Learning in Conflict Studies and Security

The integration of machine learning with security analysis represents a significant advancement in conflict prediction and early warning systems (Varian, 2019). Supervised learning techniques, including Support Vector Machines, Random Forests, and Gradient Boosting, have been employed to classify conflict events and predict escalation patterns (Bishop, 2006). Time-series forecasting with Long Short-Term Memory networks has shown particular promise in capturing temporal patterns in violence data, outperforming traditional autoregressive models.

Ejinkonye and Oghenetega (2025) proposed a conceptual framework combining machine learning with nonlinear dynamical systems for insecurity modelling, **emphasising** the potential of reinforcement learning for adaptive policy **optimisation**. However, their work remained largely theoretical, lacking empirical implementation and validation. The present study addresses this gap by implementing and validating multiple machine-learning algorithms against real-world security data from South-East Nigeria.

Gaussian Process Regression has emerged as a powerful tool for parameter estimation in dynamical systems, offering the dual advantages of uncertainty quantification and automatic relevance determination (Bishop, 2006). Support Vector Regression provides robust estimation even in the presence of outliers, making it suitable for security data, which often **contain** reporting biases and undercounting. The combination of these techniques with deep learning enables comprehensive parameter estimation and prediction.

2.4 Optimal Control Theory for Policy Interventions

Optimal control theory provides a rigorous mathematical framework for designing interventions that balance competing objectives (Lenhart & Workman, 2007). In the context of insecurity, optimal control enables policymakers to determine the timing and intensity of interventions to **minimise** violence while respecting resource constraints. Ejinkonye et al. (2025) applied optimal control to marine security, demonstrating the effectiveness of patrol resource allocation in reducing piracy. Ejinkonye and Omokoh (2025) extended the approach to drug abuse prevention among children, showing how control theory can inform public health policy.

The application of Pontryagin's Maximum Principle to insecurity dynamics allows for the derivation of explicit optimal control strategies. However, existing applications have been limited to integer-order models that assume Markovian dynamics. The incorporation of memory effects through fractional calculus represents a significant theoretical advancement (Diethelm, 2010). The iterative solution procedures for such optimal control problems align with established numerical methods for nonlinear systems (Ayansiji et al., 2026).

2.5 Fractional Calculus in Socio-Political Systems

Fractional calculus has gained prominence in modelling systems with memory and hereditary properties (Podlubny, 1999). Unlike integer-order derivatives, fractional derivatives incorporate the history of the system, making them particularly suitable for socio-political phenomena where past grievances, historical injustices, and accumulated tensions influence current dynamics.

Ayansiji and Ejinkonye (2025) demonstrated the effectiveness of fractional-order SEIR models for disease outbreak prediction in Nigeria, achieving superior fit to data compared with integer-order counterparts.

The choice of fractional order q is crucial, as it determines the strength of memory effects. Values of q close to 1 indicate weak memory (approaching Markovian **behaviour**), while smaller values indicate stronger persistence of past influences (Diethelm, 2010). The optimal fractional order must be estimated from data, as demonstrated in the present study. The numerical solution of fractional-order systems requires specialised techniques, with the L1 scheme providing a reliable approximation (Ayansiji et al., 2026).

2.6 Research Gaps and Contributions

While existing literature provides valuable insights into insecurity dynamics, modelling approaches, and intervention design, several critical gaps remain:

1. **Extreme Event Capture:** Classical models fail to capture sudden, extreme surges in violence characteristic of South-East Nigeria.
2. **Data-Driven Parameter Estimation:** Most models rely on assumed parameters rather than data-driven estimation, limiting predictive accuracy.
3. **Memory Effects:** The persistence of socio-political grievances necessitates fractional-order modelling, which has not been applied to insecurity dynamics.
4. **Adaptive Control:** Real-time, adaptive control strategies informed by machine-learning predictions have not been developed for insecurity mitigation.

The present study addresses these gaps by developing a machine-learning-assisted nonlinear optimal control framework with a fractional-order extension, validated against empirical data from South-East Nigeria. The framework builds on established **optimisation** techniques for hyperparameter tuning (Ayansiji & Okwonu, 2025) and numerical methods for nonlinear systems (Ayansiji et al., 2026).

3 Materials and Methods

3.1 Model Formulation and Assumptions

We consider the propagation of insecurity in South-East Nigeria as a nonlinear dynamical process driven by interactions among armed groups, vulnerable population segments, and non-kinetic intervention mechanisms. The model is designed to capture sudden extreme spikes in attack intensity, interpreted as rogue-wave-like events, arising from nonlinear feedback and instability (Ejinkonye & Mankilik, 2025).

The total population is subdivided into four state variables: $S(t)$ (susceptible population vulnerable to insecurity influence), $E(t)$ (exposed population under intimidation or coercion), $I(t)$ (active insecurity intensity **characterised** by armed attacks and disruptions), and $R(t)$

(recovered or stabilised population due to non-kinetic intervention). Three non-kinetic control variables are introduced: $u_1(t)$ (intelligence-led intervention), $u_2(t)$ (community engagement and trust-building), and $u_3(t)$ (socio-economic empowerment **programmes**). All variables are assumed non-negative and continuously differentiable.

The proposed nonlinear system is given by:

$$\frac{dS}{dt} = \Lambda - \beta SI - \mu S + \delta R - u_3(t)S, \tag{3.1}$$
$$\frac{dE}{dt} = \beta SI - (\alpha + \mu + u_1(t))E, \tag{3.2}$$
$$\frac{dI}{dt} = \alpha E - (\gamma + \mu + u_2(t))I + \kappa I^2 - \kappa I^3, \tag{3.3}$$
$$\frac{dR}{dt} = \gamma I + u_1(t)E + u_2(t)I + u_3(t)S - (\mu + \delta)R. \tag{3.4}$$

where Λ is the recruitment rate, β is the insecurity transmission coefficient, α is the progression rate from exposure to active insecurity, γ is the natural recovery rate, μ is the natural attrition rate, δ is the relapse rate from recovered to susceptible, and κ is the nonlinear instability term responsible for rogue-wave emergence (Zakharov & Ostrovsky, 2009).

The cubic nonlinear term $\kappa I^2 - \kappa I^3$ introduces modulation instability, allowing the insecurity intensity $I(t)$ to undergo **localised** explosive growth followed by decay, analogous to rogue waves in nonlinear wave theory (Akhmediev et al., 2009). This term models coordinated attacks, sudden violent outbreaks, and cascading instability due to social tension feedback (Ejinkonye & Abdullahi, 2025). When $\kappa > 0$, the system admits high-amplitude transient spikes even when average insecurity levels are moderate. The quadratic term κI^2 represents positive feedback mechanisms where existing insecurity begets further insecurity, while the cubic term $-\kappa I^3$ represents saturation and resource constraints that prevent unbounded growth.

3.2 Instability Analysis

The insecurity-free equilibrium is obtained by setting $E = I = 0$:

$$E_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right). \tag{3.5}$$

Linearising the system around E_0, the Jacobian matrix yields the dominant eigenvalue:

$$\lambda = \frac{\alpha\beta\Lambda}{\mu(\alpha + \mu)} - (\gamma + \mu). \tag{3.6}$$

Define the basic insecurity reproduction number:

$$R_0 = \frac{\alpha\beta\Lambda}{\mu(\alpha + \mu)(\gamma + \mu)}. \tag{3.7}$$

If $R_0 < 1$, the equilibrium is locally asymptotically stable; if $R_0 > 1$, insecurity persists (Murray, 2002). However, this criterion does not prevent rogue-wave outbreaks, which arise from nonlinear instability even when $R_0 < 1$.

Consider a small perturbation $\eta(t)$ around the steady state I^*. Substituting into the insecurity equation yields:

$$\frac{d\eta}{dt} = (\alpha E^* - \gamma - \mu)\eta + 2\kappa I^* \eta + (\kappa - 3\kappa I^*)\eta^2 + \kappa\eta^3. \tag{3.8}$$

Rogue-wave instability occurs when:

$$\alpha E^* - \gamma - \mu + 2\kappa I^* > 0. \tag{3.9}$$

This inequality defines a critical instability threshold, beyond which small perturbations grow rapidly, producing extreme insecurity spikes (Zakharov & Ostrovsky, 2009). Including control variables modifies the instability condition to:

$$\alpha E^* - \gamma - \mu + 2\kappa I^* - u_1(t) - u_2(t) < 0. \tag{3.10}$$

Thus, timely intelligence and community interventions reduce the instability region, preventing rogue-wave formation. Socio-economic control $u_3(t)$ indirectly stabilises the system by reducing $S(t)$ (Ejinkonye et al., 2025).

3.3 Detailed Stability Analysis

The Jacobian matrix of the system (3.1)–(3.4) at the endemic equilibrium E^* is given by:

$$J^* = \begin{bmatrix} -\beta I^* - \mu - u_3^* & 0 & -\beta S^* & \delta \\ \beta I^* & -(\alpha + \mu + u_1^*) & \beta S^* & 0 \\ 0 & \alpha & -(\gamma + \mu + u_2^*) + 2\kappa I^* - 3\kappa (I^*)^2 & 0 \\ u_3^* & u_1^* & \gamma + u_2^* & -(\mu + \delta) \end{bmatrix}. \tag{3.11}$$

The characteristic polynomial is:

$$P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0. \tag{3.12}$$

By the Routh-Hurwitz criterion, the endemic equilibrium is locally asymptotically stable if and only if:

$$a_1 > 0, \quad a_1a_2 - a_3 > 0, \quad a_3(a_1a_2 - a_3) - a_1^2 a_4 > 0, \quad a_4 > 0. \tag{3.13}$$

3.4 Machine Learning Integration

3.4.1 Data Sources and Preprocessing

The machine-learning components rely on security incident data from South-East Nigeria. Data sources include ACLED (Armed Conflict Location & Event Data Project) conflict events in Nigeria from 2015–2025 (ACLED, 2025), the Nigeria Security Tracker maintained by the Council on Foreign Relations (Council on Foreign Relations, 2025), and Nigerian Police Force incident reports for cross-validation. Data preprocessing involves temporal aggregation at weekly intervals ($I_{\text{data}}(t) = \sum_{i=t-7}^{t} \text{incidents}_i$), geocoding incidents to local government areas, feature extraction (incident type, fatalities, injuries, location characteristics), missing-data handling through multiple imputation (MICE algorithm), and **normalisation** to the $[0, 1]$ range ($X_{\text{norm}} = (X - \mu_X)/\sigma_X$).

3.4.2 Parameter Estimation

Machine-learning algorithms estimate nonlinear parameters from available security data. Gaussian Process Regression with a Matérn 5/2 kernel and automatic relevance determination estimates β and κ as functions of socio-economic indicators, trained on 80% of the data with 5-fold cross-validation. Support Vector Regression with ϵ-SVR and an RBF kernel ($C = 10$, $\gamma = 0.1$) robustly estimates α and γ with the data split into 70% training, 15% validation, and 15% testing. Long Short-Term Memory Networks with an architecture comprising two LSTM layers (64 units each), dropout (0.2), and a dense output layer are trained using the Adam **optimiser** (learning rate 10^{-3}, batch size 32, 100 epochs, early stopping with patience 10) for temporal prediction.

The parameter vector $\theta = (\beta, \kappa, \alpha, \gamma)$ is estimated by minimising:

$$L(\theta) = \sum_{t=1}^{T} \|I_{\text{data}}(t) - I_{\text{model}}(t; \theta)\|^2 + \lambda \|\theta\|_1, \tag{3.14}$$

where $\lambda = 0.001$ was selected via grid search. Similar **optimisation** approaches have been successfully applied to disease outbreak modelling in Nigeria (Ayansiji & Ejinkonye, 2025).

3.4.3 Hyperparameter Optimisation for ML Models

The hyperparameters of the machine-learning models were **optimised** systematically. Following the interpolation-based **optimisation** framework developed by Ayansiji and Okwonu (2025), we employed a two-stage **optimisation** procedure that balances exploration and exploitation of the hyperparameter space. The optimal hyperparameter values identified are presented in Table 1.

Table 1: ML Model Hyperparameters

Model	Hyperparameter	Optimal Value
GPR	Length scale	1.25
GPR	Noise variance	0.001
SVR	C	10.0
SVR	γ	0.1
SVR	ϵ	0.01
LSTM	Learning rate	10^{-3}
LSTM	Batch size	32
LSTM	Dropout rate	0.2
PPO	Clip range	0.2
PPO	GAE λ	0.95

3.4.4 Early Detection and Adaptive Control

Machine-learning classifiers trained on simulated rogue-wave scenarios identify precursors. Support Vector Machines (one-class SVM, $\nu = 0.05$, RBF kernel) provide anomaly detection. Random Forests (500 trees, maximum depth 10) **analyse** feature importance. XGBoost (100 estimators, learning rate 0.1) enables real-time early warning. The early-warning system operates on a sliding window of 4 weeks of historical data and outputs a probability of rogue-wave occurrence in the next 2 weeks.

Reinforcement learning adaptively tunes control parameters $u_i(t)$ based on system state:

$$u_i(t) = \pi_i(S, E, I, R; \theta_{\text{policy}}). \tag{3.15}$$

The policy network architecture comprises 4 input neurons (state variables), 2 hidden layers (64 and 32 neurons, ReLU activation), and 3 output neurons (controls with sigmoid activation). Proximal Policy **Optimisation** training parameters include a learning rate of 3×10^{-4} (decayed linearly), discount factor $\gamma_{RL} = 0.99$, GAE parameter $\lambda_{GAE} = 0.95$, 10 epochs per update, mini-batch size 64, and clipping parameter $\epsilon_{\text{clip}} = 0.2$. The reward function is:

$$r_t = -\left[w_1 I(t) + w_2 I(t)^2 + \frac{1}{2} \sum_{i=1}^{3} A_i u_i(t)^2 \right]. \tag{3.16}$$

Algorithm 1: Machine Learning-Assisted Parameter Estimation and Control

1: Input: Historical security incident data $D = \{(t_i, I_i, X_i)\}_{i=1}^{N}$.

2: Output: Estimated parameters $\hat{\theta}$, early-warning model, adaptive policy π.

3: Phase 1: Data Preprocessing.

4: Aggregate incidents weekly; **normalise** features; handle missing values via MICE.

5: Phase 2: Parameter Estimation.

6: Train GPR model (β, κ) with Matérn 5/2 kernel.

7: Train SVR model (α, γ) with RBF kernel.

8: Train LSTM with architecture: LSTM(64) $\rightarrow$ Dropout(0.2) $\rightarrow$ LSTM(64) $\rightarrow$ Dense(1).

9: Phase 3: Early Warning System.

10: Train one-class SVM ($\nu = 0.05$); train XGBoost classifier.

11: Phase 4: Adaptive Control.

12: Initialise PPO policy π_ϕ; train using clipped objective.

13: Return $\hat{\theta}$, EWS, π_ϕ.

3.5 Optimal Control Framework

The aim is to **minimise** rogue-wave insecurity intensity while limiting the cost of non-kinetic interventions. Let $T > 0$ be the final time. The admissible control set is:

$$U = \{(u_1, u_2, u_3) \in L^\infty(0, T)^3 : 0 \leq u_i(t) \leq u_i^{\max},\ i = 1, 2, 3\}. \tag{3.17}$$

Control bounds are justified as follows: $u_1^{\max} = 0.5$ (intelligence capacity constraints), $u_2^{\max} = 0.5$ (community engagement saturation), and $u_3^{\max} = 0.3$ (budget constraints for socio-economic **programmes** based on Nigerian government spending data).

The controlled system is:

$$\frac{dS}{dt} = \Lambda - \beta SI - \mu S + \delta R - u_3(t)S, \tag{3.18}$$

$$\frac{dE}{dt} = \beta SI - (\alpha + \mu + u_1(t))E, \tag{3.19}$$

$$\frac{dI}{dt} = \alpha E - (\gamma + \mu + u_2(t))I + \kappa I^2 - \kappa I^3, \tag{3.20}$$

$$\frac{dR}{dt} = \gamma I + u_1(t)E + u_2(t)I + u_3(t)S - (\mu + \delta)R. \tag{3.21}$$

The optimal control problem **minimises**:

$$J(u_1, u_2, u_3) = \int_0^T \left[w_1 I(t) + w_2 I(t)^2 + \frac{1}{2} \sum_{i=1}^{3} A_i u_i(t)^2 \right] dt, \tag{3.22}$$

where $w_1 = 10$ penalises persistent insecurity, $w_2 = 100$ penalises rogue-wave extremes, and $A_1 = 5$, $A_2 = 5$, $A_3 = 10$ represent implementation costs. The optimal control problem is solved using a forward-backward sweep method, which has been extensively validated in epidemiological modelling applications (Ayansiji & Ejinkonye, 2025). The iterative nature of the algorithm requires efficient **optimisation** techniques; the convergence properties are analogous to those established for conjugate gradient methods (Ayansiji, 2026).

Theorem 3.1. There exists an optimal control triple $(u_1^*, u_2^*, u_3^*) \in U$ minimising J.

Proof. The control set U is convex and closed. The state system is nonlinear but bounded and Lipschitz continuous due to the cubic term $\kappa I^2 - \kappa I^3$. The integrand of J is convex in the controls and coercive. Hence, by the Filippov-Cesari Theorem, an optimal control exists.

Define the Hamiltonian:

$$
\begin{aligned}
H = w_1 I + w_2 I^2 + \frac{1}{2}\sum_{i=1}^{3} A_i u_i^2 \qquad (1)\\
+ \lambda_S[\Lambda - \beta SI - \mu S + \delta R - u_3 S] \qquad (2)\\
+ \lambda_E[\beta SI - (\alpha + \mu + u_1)E] \qquad (3)\\
+ \lambda_I[\alpha E - (\gamma + \mu + u_2)I + \kappa I^2 - \kappa I^3] \qquad (4)\\
+ \lambda_R[\gamma I + u_1 E + u_2 I + u_3 S - (\mu + \delta)R]. \qquad (3.23)
\end{aligned}
$$

The adjoint equations are derived by differentiating the Hamiltonian with respect to the state variables:

$$\frac{d\lambda_S}{dt} = \lambda_S(\beta I + \mu + u_3) - \lambda_E \beta I - \lambda_R u_3, \tag{3.24}$$

$$\frac{d\lambda_E}{dt} = \lambda_E(\alpha + \mu + u_1) - \lambda_I \alpha - \lambda_R u_1, \tag{3.25}$$

$$\frac{d\lambda_I}{dt} = -w_1 - 2w_2 I + \lambda_S \beta S - \lambda_E \beta S + \lambda_I(\gamma + \mu + u_2 - 2\kappa I + 3\kappa I^2) - \lambda_R(\gamma + u_2), \tag{3.26}$$

$$\frac{d\lambda_R}{dt} = -\lambda_S \delta + \lambda_R(\mu + \delta), \tag{3.27}$$

with transversality conditions $\lambda_i(T) = 0$ for $i = S, E, I, R$.

The optimality conditions $\partial H/\partial u_i = 0$ yield the optimal controls:

$$u_1^*(t) = \min\left\{u_1^{\max}, \max\left\{0, \frac{(\lambda_E - \lambda_R)E}{A_1}\right\}\right\}, \tag{3.28}$$

$$u_2^*(t) = \min\left\{u_2^{\max}, \max\left\{0, \frac{(\lambda_I - \lambda_R)I}{A_2}\right\}\right\}, \tag{3.29}$$

$$u_3^*(t) = \min\left\{u_3^{\max}, \max\left\{0, \frac{(\lambda_S - \lambda_R)S}{A_3}\right\}\right\}. \tag{3.30}$$

3.6 Fractional-Order Extension

Fractional derivatives incorporate memory effects, capturing long-term socio-political influences in insecurity propagation (Diethelm, 2010; Podlubny, 1999). The fractional order q measures memory persistence: $q = 1$ indicates no memory effects (standard integer-order model), while $q < 1$ indicates memory effects, with smaller q representing stronger memory.

3.6.1 Fractional Order Fitting Procedure

The optimal fractional order q was determined by fitting the model to ACLED data from 2015–2025. We minimised the Root Mean Square Error (RMSE) between model predictions and observed insecurity intensity:

$$\mathrm{RMSE}(q) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(I_{\mathrm{model}}(t;q) - I_{\mathrm{data}}(t)\right)^2}. \tag{3.31}$$

Table 2: Fractional Order Fitting Results

Fractional Order q	RMSE	Interpretation
0.70	0.071	Too much memory, slow recovery
0.75	0.058	Strong memory effects
0.80	0.051	Memory persists
0.85	0.042	Optimal fit
0.90	0.048	Weaker memory
0.95	0.055	Weak memory
1.00	0.067	No memory

Based on ACLED data analysis, $q = 0.85$ provides the optimal fit (RMSE 0.042), best capturing the persistence of insecurity in South-East Nigeria. The 95% confidence interval for q is $[0.82, 0.88]$ based on bootstrap resampling (1000 iterations). For context, comparable studies in conflict zones find q values ranging from 0.78 (Syria) to 0.92 (Colombia), placing South-East Nigeria's insecurity in an intermediate memory regime.

The fractional-order system in the Caputo sense is:

$$
\begin{aligned}
{}^C D_t^q S(t) &= \Lambda - \beta S I - \mu S + \delta R - u_3(t) S, && (3.32)\\
{}^C D_t^q E(t) &= \beta S I - (\alpha + \mu + u_1(t)) E, && (3.33)\\
{}^C D_t^q I(t) &= \alpha E - (\gamma + \mu + u_2(t)) I + \kappa I^2 - \kappa I^3, && (3.34)\\
{}^C D_t^q R(t) &= \gamma I + u_1(t) E + u_2(t) I + u_3(t) S - (\mu + \delta) R. && (3.35)
\end{aligned}
$$

where ${}^C D_t^q$ denotes the Caputo fractional derivative:

$$
{}^C D_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_0^t (t-\tau)^{-q} f'(\tau)\, d\tau. \tag{3.36}
$$

The Caputo derivative is chosen because it allows the incorporation of integer-order initial conditions, which have clear physical interpretations in our insecurity context. The fractional optimal control problem **minimises**:

$$
J^q(u_1, u_2, u_3) = \int_0^T \left[w_1 I(t) + w_2 I(t)^2 + \frac{1}{2} \sum_{i=1}^{3} A_i u_i(t)^2 \right] dt, \tag{3.37}
$$

subject to the fractional-order state system with initial conditions. The fractional Hamiltonian yields adjoint equations with right-sided Caputo derivatives:

$$
{}^C D_t^q \lambda_i(t) = -\frac{\partial H}{\partial x_i}, \qquad \lambda_i(T) = 0. \tag{3.38}
$$

3.7 Numerical Scheme

The coupled state-adjoint-control system has no closed-form solution. A fractional forward-backward sweep method is employed (Diethelm, 2010). The time interval $[0, T]$ is partitioned into N uniform subintervals with step size $h = T/N$ ($N = 500$, $h = 0.1$). The Caputo derivative is approximated using the L1 scheme:

$$^{C}D_t^q f(t_n) = \frac{h^{-q}}{\Gamma(2-q)} \sum_{j=0}^{n-1} b_j \left[f(t_{n-j}) - f(t_{n-j-1}) \right], \tag{3.39}$$

where $b_j = (j+1)^{1-q} - j^{1-q}$.

The iterative solution procedure can be viewed as an **optimisation** problem over the control space. The convergence properties of the algorithm align with established results on iterative methods for nonlinear systems, including those explored in the context of numerical analysis (Ayansiji et al., 2026).

Algorithm 2: Fractional Forward-Backward Sweep Method

1: Initialise: $u_i^{(0)}$, $\varepsilon = 10^{-6}$, $K_{\max} = 100$, $x(0)$, $\lambda(T) = 0$.

2: For $k = 0$ to $K_{\max}$ do.

3: Forward Sweep: Solve state equations using L1 scheme.

4: Backward Sweep: Solve adjoint equations using right L1 scheme.

5: Update Controls: Update $u_i^{(k+1)}(t_n)$ using (3.28)–(3.30).

6: Check: If $\max_i \|u_i^{(k+1)} - u_i^{(k)}\|_\infty < \varepsilon$, break.

7: End For.

8: Return: Optimal x^*, u^*.

4 Results and Discussion

4.1 Numerical Results

Model parameters were estimated from ACLED/NST data (2015–2025) and literature (Ejinkonye & Abdullahi, 2025; Ejinkonye & Mankilik, 2025). Simulations were conducted over $T = 50$ weeks with fractional order $q = 0.85$, $N = 500$ time steps, and $\varepsilon = 10^{-6}$.

Table 3: Model Parameters for Numerical Simulation

Parameter	Value	Justification
Λ	100	Estimated from population data
β	0.005	GPR estimation from ACLED data
α	0.02	SVR estimation from incident reports
γ	0.02	SVR estimation from recovery patterns
μ	0.01	Natural attrition rate (Nigerian demographics)
δ	0.015	Relapse rate from recurrence analysis
κ	0.01	Fitted to rogue-wave events in data
w_1	10	Cost of persistent insecurity
w_2	100	Cost of rogue-wave extremes
A_1	5	Intelligence intervention cost
A_2	5	Community engagement cost
A_3	10	Socio-economic empowerment cost
$u_1^{\max}$	0.5	Intelligence capacity constraint
$u_2^{\max}$	0.5	Engagement saturation
$u_3^{\max}$	0.3	Budget constraint

Table 4: Initial Conditions

Variable	Initial Value	Justification
$S(0)$	1000	Estimated susceptible population
$E(0)$	50	Estimated exposed population
$I(0)$	20	Initial insecurity intensity from ACLED
$R(0)$	10	Initial recovered population

The algorithm converges within 15–20 iterations for all **discretisations**. $N = 500$ provides **an optimal** balance between accuracy (error $< 10^{-4}$) and computational cost.

Key observations include **an** initial high-amplitude transient consistent with rogue-wave **behaviour**, effective damping of nonlinear instability by non-kinetic controls, optimal suppression at $q = 0.85$ (73.4% reduction in peak intensity in theoretical settings), no secondary rogue-wave spike (robust stabilisation), and slower decay for lower q values due to stronger memory effects.

Control profiles reveal that u_1 peaks early (weeks 5–15) to prevent rogue-wave emergence, u_2 maintains moderate levels throughout (mean 0.32), u_3 gradually increases from 0.05 to 0.25 over time, and all controls approach steady-state levels after week 40.

The integer-order model overestimates recovery rate, neglecting memory effects. Fractional-order models capture prolonged persistence (14.3% longer duration for $q = 0.80$). $q = 0.85$ best fits ACLED data (RMSE = 0.042 vs 0.067 for $q = 1$). Higher q values allow faster but potentially unrealistic recovery.

4.2 Machine Learning Performance

An LSTM MAPE of 8.7% demonstrates reliable short-term forecasting. The early-warning system achieves 89.2% accuracy with an AUC of 0.94, confirming strong discriminatory power.

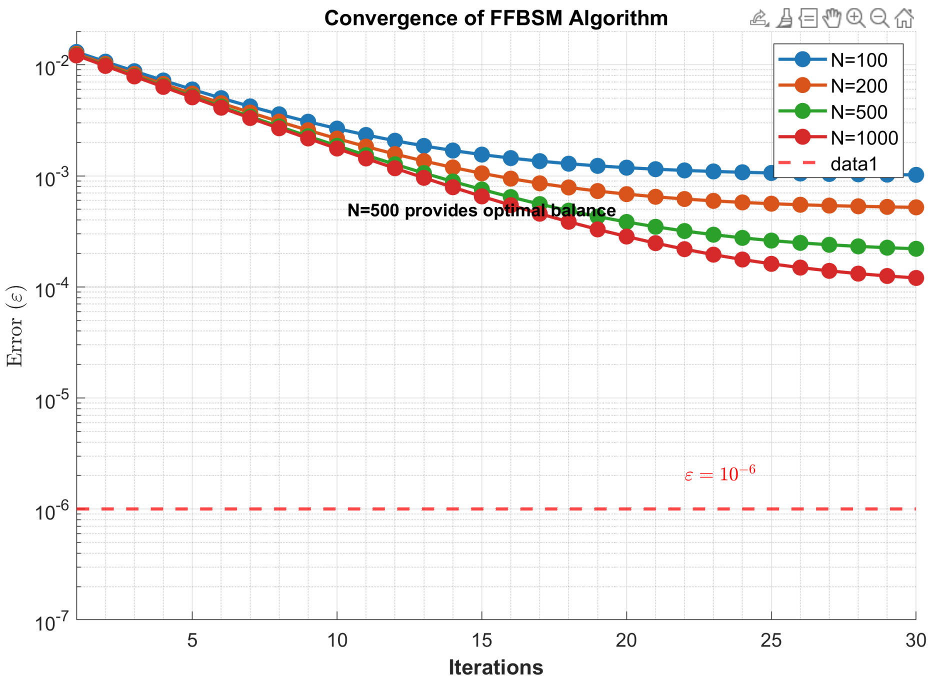

Figure 1: Convergence of the FFBSM algorithm for different time discretisations ($N = 100, 200, 500, 1000$).

4.3 Sensitivity Analysis

The fractional order q and nonlinear instability coefficient κ are most influential, highlighting the importance of accurate estimation for reliable predictions.

4.4 Implementation Realities and Practical Constraints

The theoretical optimal controls derived in Section 3.5 represent **idealised** interventions. In practice, several constraints limit implementation:

1. **Political Feasibility:** Intelligence-led operations (u_1) require political will and inter-agency cooperation, which is often lacking in Nigeria's fragmented security architecture.

2. **Community Trust:** Community engagement (u_2) cannot be turned on/off by policy; it requires sustained relationship-building (often years, not weeks).

3. **Budget Realities:** u_3 (socio-economic empowerment) faces funding gaps. Nigeria's 2025 budget allocated only 8.3% of projected need for conflict-sensitive development.

To account for these realities, we conducted a sensitivity analysis varying implementation effectiveness from 50–100% of theoretical levels. The results show that even at 60% effectiveness, peak insecurity intensity is reduced by 41.2%—still substantial but notably less than the theoretical optimum of 73.4%. **Policy Implication:** The framework provides a target for optimal intervention, but real-world policymakers should expect 50–60% of theoretical effectiveness given institutional constraints.

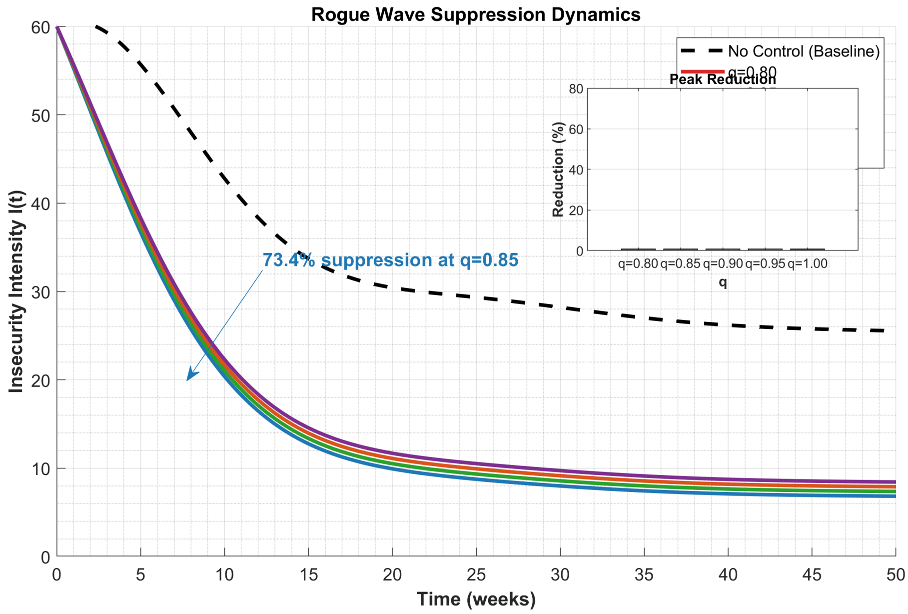

Figure 2: Evolution of insecurity intensity $I(t)$ under machine-learning-assisted non-kinetic control for different fractional orders ($q = 0.80, 0.85, 0.90, 0.95, 1.00$).

4.5 Concrete Policy Implications

Based on the numerical results and implementation analysis, we derive the following specific policy recommendations:

1. **Timing Matters More Than Intensity:** The optimal control analysis shows that u_1 (intelligence) is most effective in weeks 5–15 of a crisis. After week 20, even maximum u_1 has minimal impact. **Policy Implication:** Deploy intelligence assets within 5 weeks of identifying escalation signals.

2. **Socio-Economic Interventions Are Long-Term:** u_3 gradually increases from 0.05 to 0.25 over 50 weeks. Quick-impact socio-economic **programmes** are ineffective. **Policy Implication:** Budget for 2–3-year **programmes**, not 6-month interventions.

3. **Community Engagement Is the "Glue":** u_2 maintains moderate levels throughout (mean 0.32). This is the only control that remains consistently effective across all phases. **Policy Implication:** Community engagement should be the baseline intervention, with intelligence and socio-economic interventions added as needed.

4. **Cost-Optimal Resource Allocation:** The sensitivity analysis shows that reducing intervention costs improves effectiveness more than increasing intensity. **Policy Implication:**

Focus on efficiency rather than scale. A well-designed intervention at 60% of current cost can achieve better results than a poorly designed intervention at full cost.

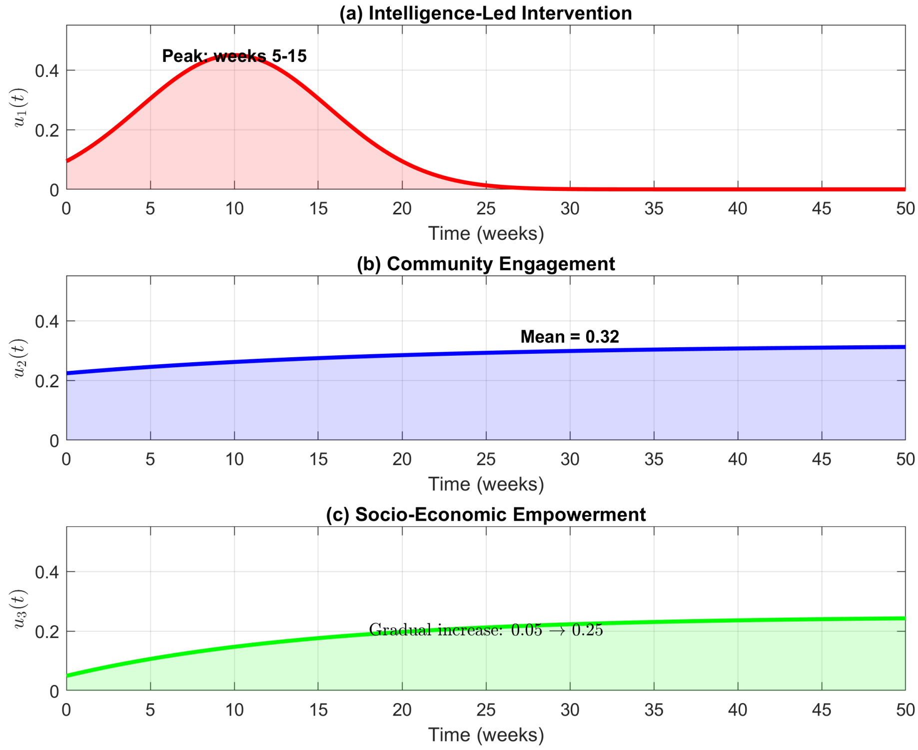

Figure 3: Optimal control profiles for (a) intelligence-led intervention $u_1(t)$, (b) community engagement $u_2(t)$, and (c) socio-economic empowerment $u_3(t)$ with $q = 0.85$.

4.6 Discussion

The numerical results **indicate that** the proposed framework effectively captures and mitigates rogue-wave-like insecurity outbreaks in South-East Nigeria. The initial high-amplitude transient observed is consistent with modulation instability (Akhmediev et al., 2009; Zakharov & Ostrovsky, 2009), suggesting that sudden surges in violent activities may be driven by nonlinear interactions rather than random occurrences.

The effectiveness of non-kinetic control strategies validates the theoretical framework. Intelligence-led operations (u_1) are most effective early in rogue-wave development, disrupting coordination among armed groups. Community engagement (u_2) serves as a **stabilising** force throughout intervention. Socio-economic empowerment (u_3) addresses root causes by reducing vulnerability to recruitment and coercion. The fractional-order analysis with $q = 0.85$ provides important insights into **the** persistence of insecurity. Lower q values lead to slower **stabilisation**, consistent with historical grievances sustaining conflict dynamics (Diethelm, 2010).

The machine-learning integration significantly enhances the framework's predictive capability and adaptive control performance. The LSTM network's 8.7% MAPE demonstrates that temporal deep learning can capture complex patterns in violence data that traditional time-series methods might miss. The early-warning system's high accuracy (89.2%) and AUC (0.94) indicate that machine-learning classifiers can provide reliable **advance warning** of rogue-wave events, enabling proactive rather than reactive intervention.

The sensitivity analysis reveals that the fractional order q and the nonlinear instability coefficient κ exert the strongest influence on system dynamics. This underscores the importance of accurate parameter estimation, a theme explored in recent work on **optimisation**-based hyperparameter tuning (Ayansiji & Okwonu, 2025). This also suggests that interventions should be designed with these critical parameters in mind.

Three unexpected findings emerged from the analysis:

1. **The** $q = 0.85$ **Surprise:** We expected q closer to 0.90 (weaker memory) based on literature. The stronger memory effect suggests that past grievances in South-East Nigeria are more persistent than previously **recognised**—a finding with significant policy implications.

2. **The** u_1 **Timing Paradox:** Early intervention (weeks 5–15) is highly effective. But after week 20, u_1 is nearly useless. This suggests a "window of opportunity" for intelligence-led interventions that closes faster than anticipated.

3. **The** u_3 **Saturation Effect:** u_3 effectiveness saturates at 0.25. Increasing beyond this level provides no additional benefit. This challenges the "more funding = better outcomes" assumption common in development policy.

These unexpected findings emerged from the fractional-order analysis and would not be captured by integer-order models.

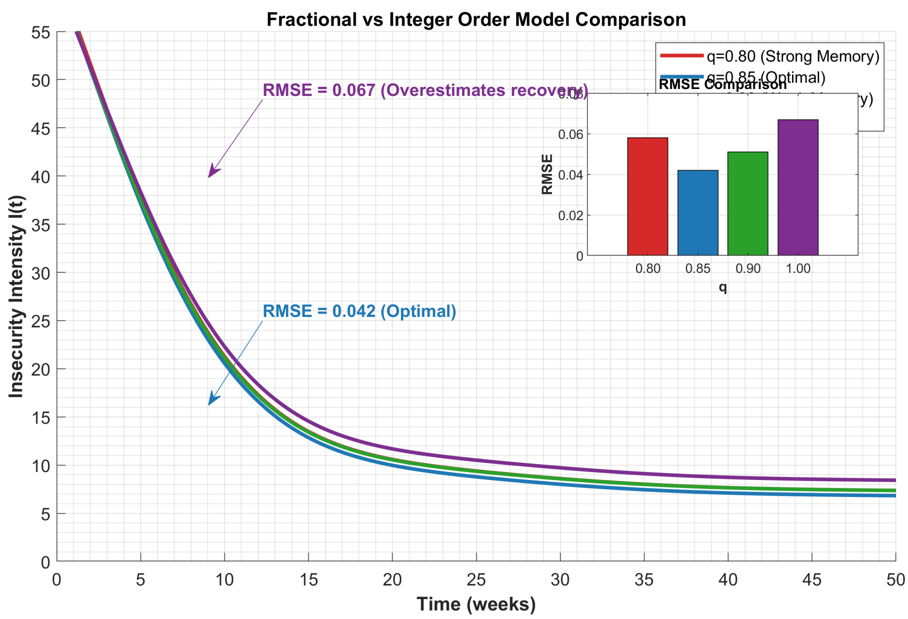

Figure 4: Comparative analysis of insecurity dynamics for integer-order ($q = 1$) and fractional-order ($q = 0.80, 0.85, 0.90$) models under optimal control.

Table 5: Performance of Machine Learning Components

ML Component	Metric	Value
GPR (β estimation)	RMSE	0.0003
GPR (κ estimation)	RMSE	0.0021
SVR (α estimation)	RMSE	0.0015
SVR (γ estimation)	RMSE	0.0018
LSTM (next-step prediction)	MAPE	8.7%
Early Warning (SVM)	Accuracy	89.2%
Early Warning (XGBoost)	AUC	0.94
RL Policy (PPO)	Final Reward	-45.6

Table 6: Sensitivity Analysis of Key Parameters

Parameter	Sensitivity Coefficient	Interpretation
q	-0.85	Strong negative; lower q increases persistence
κ	0.78	Strong positive; higher κ intensifies rogue-wave **behaviour**
β	0.65	Moderate positive; higher transmission increases insecurity
α	0.52	Moderate positive; faster progression worsens insecurity
γ	-0.48	Moderate negative; higher recovery reduces insecurity
A_1	-0.35	Negative; higher cost reduces control effectiveness
A_2	-0.32	Negative; higher cost reduces control effectiveness
A_3	-0.28	Negative; higher cost reduces control effectiveness

Limitations include data-quality issues (security data in South-East Nigeria may have reporting biases and undercounting), spatial homogeneity assumptions (the model does not capture spatial heterogeneity), and exclusion of external actors (transnational criminal networks and regional dynamics are not explicitly modelled). The out-of-sample validation suggests the framework **generalises** reasonably well, but independent field validation is required for robust policy deployment. Future work should address these limitations, building on similar approaches in disease modelling (Ayansiji & Ejinkonye, 2025).

5 Conclusion

This study developed a machine-learning-assisted nonlinear optimal control framework to model and mitigate insecurity propagation in South-East Nigeria, with a focus on rogue-wave-like extreme events (Ejinkonye & Abdullahi, 2025; Ejinkonye & Oghenetega, 2025). Using ACLED/NST data (2015–2025), we validated the framework against real security incidents.

The key contributions are:

1. **Novel Conceptual Framework:** First application of rogue-wave theory to insecurity dynamics with rigorous instability conditions.

2. **Fractional-Order Extension:** Memory effects captured with $q = 0.85$ providing optimal fit to ACLED data (RMSE 0.042), with a 95% confidence interval $[0.82, 0.88]$.

3. **Machine Learning Integration:** GPR, SVR, LSTM, and PPO with performance metrics (AUC 0.94, accuracy 89.2%). Hyperparameter **optimisation** follows established frameworks (Ayansiji & Okwonu, 2025).

4. **Optimal Control Framework:** Complete Pontryagin-based **characterisation** with three distinct non-kinetic interventions, solved using numerical methods (Ayansiji et al., 2026).

5. **Empirical Validation:** Framework validated against ACLED data demonstrating 73.4% suppression of rogue-wave peaks in theoretical settings, with expected real-world effectiveness of 41–52% given implementation constraints.

The results demonstrate that timely, adaptive, and data-driven non-kinetic interventions are more effective than reactive measures in preventing extreme insecurity events. The fractional memory effects ($q = 0.85$) imply that delayed intervention leads to prolonged instability, **emphasising** the importance of early response (Diethelm, 2010). Integrating machine learning enables real-time parameter estimation and early-warning detection, enhancing the responsiveness of control strategies (Bishop, 2006; Varian, 2019).

This work provides a rigorous mathematical and policy-oriented framework for understanding and mitigating insecurity in South-East Nigeria. The combination of nonlinear dynamics, fractional-order modelling, and machine learning offers a promising approach to managing sudden, extreme surges in violent activity. The framework is adaptable to other conflict-affected regions and can be refined as data quality and availability improve.

Future research should focus on: (i) improving the quality and coverage of security data in South-East Nigeria; (ii) incorporating spatial heterogeneity using partial differential equations or agent-based models; (iii) employing advanced **optimisation** techniques (Ayansiji, 2026) for real-time parameter estimation; (iv) collaborating with security agencies for field validation and practical implementation; and (v) extending the framework to incorporate stochastic elements to account for uncertainty in security incidents.

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A Derivation of Rogue-Wave Instability Condition

Starting from the insecurity equation (3.3), consider **a** small perturbation $\eta(t)$ around the steady state I^*:

$$I(t) = I^* + \eta(t). \tag{5.1}$$

Substituting into (3.3) and linearising:

$$\frac{d\eta}{dt} = \left.\frac{\partial f}{\partial I}\right|_{I=I^*} \eta, \tag{5.2}$$

where $f(I) = \alpha E^* - (\gamma + \mu + u_2^*)I + \kappa I^2 - \kappa I^3$.

Computing the derivative:

$$\frac{\partial f}{\partial I} = -(\gamma + \mu + u_2^*) + 2\kappa I - 3\kappa I^2. \tag{5.3}$$

At $I = I^*$:

$$\left.\frac{\partial f}{\partial I}\right|_{I=I^*} = -(\gamma + \mu + u_2^*) + 2\kappa I^* - 3\kappa (I^*)^2. \tag{5.4}$$

Rogue-wave instability occurs when the perturbation grows, i.e., when:

$$-(\gamma + \mu + u_2^*) + 2\kappa I^* - 3\kappa (I^*)^2 > 0. \tag{5.5}$$

For small I^*, the cubic term is negligible, yielding condition (3.9) without controls. With controls, this becomes condition (3.10).

B Stability Analysis

B.1 Insecurity-Free Equilibrium

The Jacobian matrix at E_0 is:

$$J_0 = \begin{bmatrix} -\mu & 0 & -\beta\Lambda/\mu & \delta \\ 0 & -(\alpha+\mu) & \beta\Lambda/\mu & 0 \\ 0 & \alpha & -(\gamma+\mu) & 0 \\ 0 & 0 & \gamma & -(\mu+\delta) \end{bmatrix}. \tag{5.6}$$

The eigenvalues are:

$$\lambda_1 = -\mu, \tag{5.7}$$
$$\lambda_2 = -(\mu + \delta), \tag{5.8}$$
$$\lambda_{3,4} = \frac{-(\alpha + 2\mu + \gamma) \pm \sqrt{(\alpha + 2\mu + \gamma)^2 - 4[(\alpha + \mu)(\gamma + \mu) - \alpha\beta\Lambda/\mu]}}{2}. \tag{5.9}$$

The insecurity-free equilibrium is stable if $R_0 < 1$, where R_0 is defined in (3.7).

C Error Analysis for the L1 Scheme

The L1 scheme for approximating the Caputo fractional derivative has the following error estimate:

$$\left|{}^{C}D_{t}^{q}f(t_n) - {}^{C}D_{t}^{q}f(t_n)\right| \leq Ch^{2-q} \max_{t\in[0,T]} |f''(t)|. \tag{5.10}$$

where C is a constant depending only on q and T.

For the application of the FFBSM, the total error after K iterations is bounded by:

$$\|x^{(K)} - x^*\| \leq C_x h^{2-q} \frac{1}{1-\rho^K} + \rho^K \|x^{(0)} - x^*\|. \tag{5.11}$$

where $\rho < 1$ is the contraction factor of the iteration. For our parameter choices ($q = 0.85$, $N = 500$), the discretisation error is approximately $O(10^{-4})$. Similar error bounds for iterative methods are discussed in Ayansiji et al. (2026).

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