***Original Research Article***

# ROGUE WAVE DYNAMICS IN INSECURITY PROPAGATION IN SOUTH-EAST NIGERIA: A MACHINE-LEARNING ASSISTED NONLINEAR OPTIMAL CONTROL FRAMEWORK FOR NON-KINETIC INTERVENTION

## Abstract

Insecurity in South-East Nigeria is characterized by sudden, extreme surges in violent activity that defy explanation by classical linear or epidemic-type models. This study proposes a nonlinear rogue-wave framework that captures these abrupt spikes through modulation instability arising from nonlinear interactions among armed groups, socio-economic stressors, and state responses. Using security incident data from ACLED and the Nigeria Security Tracker (2015-2025), machine-learning techniques including Gaussian Process Regression, Support Vector Regression, LSTM networks, and reinforcement learning are integrated for parameter estimation, early detection, and adaptive control tuning. An optimal non-kinetic control problem is formulated with three time-dependent controls—intelligence-led operations, community engagement, and socio-economic interventions—with optimality conditions derived via Pontryagin's Maximum Principle. A fractional-order extension incorporating memory effects is developed, yielding $q = 0.85$ as the optimal fit to ACLED data (RMSE 0.042). Numerical simulations demonstrate that the proposed strategies suppress rogue-wave insecurity spikes by 73.4% while minimizing intervention costs. The machine learning early warning system achieves 89.2% accuracy with AUC 0.94. Sensitivity analysis confirms the fractional-order parameter's critical role in modulating system dynamics. This mathematically rigorous, policy-relevant framework offers actionable insights for mitigating extreme insecurity outbreaks in South-East Nigeria through timely, data-driven non-kinetic interventions.

*Keywords: Rogue waves; Insecurity dynamics; Machine learning; Nonlinear optimal control; Non-kinetic intervention; South-East Nigeria; Fractional calculus*

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## 1 Introduction

Insecurity remains a persistent socio-political and economic challenge in many regions of the world, particularly in fragile and post-conflict environments. In South-East Nigeria, insecurity has evolved beyond sporadic violent incidents into a complex, nonlinear phenomenon characterized by sudden surges in attacks, disruptions of socio-economic activities, and erosion of public trust in governance. These abrupt escalations—often unpredictable in timing and magnitude—pose significant challenges to traditional security planning and response strategies (Onuoha, 2018).

Classical mathematical models of insecurity and conflict dynamics have largely relied on linear, compartmental, or epidemic-type frameworks, which assume smooth evolution and gradual changes in attack intensity (Hethcote, 2000; Murray, 2002). While such models provide useful baseline insights, they fail to adequately capture extreme, short-lived, high-impact events, such as coordinated attacks or sudden outbreaks of violence, frequently observed in South-East Nigeria. Recent studies have emphasized the need for nonlinear and instability-driven modelling approaches capable of describing these sharp transitions and explosive dynamics (Ejinkonye & Abdullahi, 2025; Ejinkonye provides a natural framework for incorporating such memory effects, capturing the long-term influence of past events on current dynamics (Diethelm, 2010; Podlubny, 1999). The integration of fractional-order modelling with optimal control theory offers enhanced capability for designing interventions that account for the lasting impact of socio-economic and political factors. Similar approaches have been successfully applied to disease outbreak modeling in Nigeria (Ayansiji & Ejinkonye, 2025).

This paper makes several distinct contributions that extend our previous work. While Ejinkonye and Abdullahi (2025) introduced nonlinear modelling for insecurity, the present study formalizes the rogue-wave analogy with rigorous mathematical conditions for modulation instability in the insecurity context. Unlike Ejinkonye and Oghenetega (2025), which proposed machine learning conceptually, this work implements Gaussian Process Regression, Support Vector Regression, LSTM networks, and Proximal Policy Optimization algorithms with concrete performance metrics validated against ACLED and Nigeria Security Tracker data. We extend beyond integer-order models by incorporating fractional calculus with explicit justification for fractional order values and comprehensive sensitivity analysis. Finally, this work provides the first complete Pontryagin-based optimal control characterization for insecurity dynamics with three distinct non-kinetic interventions.

Motivated by these gaps, this study develops a machine-learning-assisted nonlinear optimal control framework to investigate rogue-wave dynamics in the propagation of insecurity in South-East Nigeria. The proposed model captures sudden insecurity spikes through nonlinear instability mechanisms, employs machine learning to enhance parameter estimation and prediction, incorporates optimal non-kinetic controls to suppress extreme events efficiently, and extends the analysis to fractional-order systems to account for memory effects. By combining nonlinear dynamics, artificial intelligence, fractional calculus, and control theory, this work provides both a mathematically rigorous and policy-relevant framework for understanding and mitigating insecurity in the region.

# 2 Materials and Methods

## 2.1 Model Formulation and Assumptions

We consider the propagation of insecurity in South-East Nigeria as a nonlinear dynamical process driven by interactions among armed groups, vulnerable population segments, and non-kinetic intervention mechanisms. The model is designed to capture sudden extreme spikes in attack intensity, interpreted as rogue-wave-like events, arising from nonlinear feedback and instability (Ejinkonye & Mankilik, 2025).

The total population is subdivided into four state variables: $S(t)$ (susceptible population vulnerable to insecurity influence), $E(t)$ (exposed population under intimidation or coercion), $I(t)$ (active insecurity intensity characterized by armed attacks and disruptions), and $R(t)$ (recovered or stabilized population due to non-kinetic intervention). Three non-kinetic control variables are introduced: $u_1(t)$ (intelligence-led intervention), $u_2(t)$ (community engagement and trust-building), and $u_3(t)$ (socio-economic empowerment programs). All variables are assumed non-negative and continuously differentiable.

The proposed nonlinear system is given by:

$$\frac{dS}{dt} = \Lambda - \beta SI - \mu S + \delta R - u_3(t)S \tag{2.1}$$

$$\frac{dE}{dt} = \beta SI - (\alpha + \mu + u_1(t))E \tag{2.2}$$

$$\frac{dI}{dt} = \alpha E - (\gamma + \mu + u_2(t))I + \kappa I^2 - \kappa I^3 \tag{2.3}$$

$$\frac{dR}{dt} = \gamma I + u_1(t)E + u_2(t)I + u_3(t)S - (\mu + \delta)R \tag{2.4}$$

where $\Lambda$ is the recruitment rate, $\beta$ is the insecurity transmission coefficient, $\alpha$ is the progression rate from exposure to active insecurity, $\gamma$ is the natural recovery rate, $\mu$ is the natural attrition rate, $\delta$ is the relapse rate from recovered to susceptible, and $\kappa$ is the nonlinear instability term responsible for rogue-wave emergence (Zakharov & Ostrovsky, 2009).

The cubic nonlinear term $\kappa I^2 - \kappa I^3$ introduces modulation instability, allowing the insecurity intensity $I(t)$ to undergo localized explosive growth followed by decay, analogous to rogue waves in nonlinear wave theory (Akhmediev et al., 2009). This term models coordinated attacks, sudden violent outbreaks, and cascading instability due to social tension feedback (Ejinkonye & Abdullahi, 2025). When $\kappa > 0$, the system admits high-amplitude transient spikes even when average insecurity levels are moderate. The quadratic term $\kappa I^2$ represents positive feedback mechanisms where existing insecurity begets further insecurity, while the cubic term $-\kappa I^3$ represents saturation and resource constraints that prevent unbounded growth.

## 2.2 Instability Analysis

The insecurity-free equilibrium is obtained by setting $E = I = 0$:

$$\mathcal{E}_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right) \tag{2.5}$$

Linearizing the system around $\mathcal{E}_0$, the Jacobian matrix yields the dominant eigenvalue:

$$\lambda = \frac{\alpha\beta\Lambda}{\mu(\alpha+\mu)} - (\gamma+\mu) \tag{2.6}$$

Define the basic insecurity reproduction number:

$$\mathcal{R}_0 = \frac{\alpha\beta\Lambda}{\mu(\alpha+\mu)(\gamma+\mu)} \tag{2.7}$$

If $\mathcal{R}_0 < 1$, the equilibrium is locally asymptotically stable; if $\mathcal{R}_0 > 1$, insecurity persists (Murray, 2002). However, this criterion does not prevent rogue-wave outbreaks, which arise from nonlinear instability even when $\mathcal{R}_0 < 1$.

Consider a small perturbation $\eta(t)$ around the steady state $I^*$. Substituting into the insecurity equation yields:

$$\frac{d\eta}{dt} = (\alpha E^* - \gamma - \mu)\eta + 2\kappa I^* \eta + (\kappa - 3\kappa I^*)\eta^2 + \kappa\eta^3 \tag{2.8}$$

Rogue-wave instability occurs when:

$$\alpha E^* - \gamma - \mu + 2\kappa I^* > 0 \tag{2.9}$$

This inequality defines a critical instability threshold, beyond which small perturbations grow rapidly, producing extreme insecurity spikes (Zakharov & Ostrovsky, 2009). Including control variables modifies the instability condition to:

$$\alpha E^* - \gamma - \mu + 2\kappa I^* - u_1(t) - u_2(t) < 0 \tag{2.10}$$

Thus, timely intelligence and community interventions reduce the instability region, preventing rogue-wave formation. Socio-economic control $u_3(t)$ indirectly stabilizes the system by reducing $S(t)$ (Ejinkonye et al., 2025).

## 2.3 Detailed Stability Analysis

The Jacobian matrix of the system (1)-(4) at the endemic equilibrium $\mathcal{E}^*$ is given by:

$$J^* = \begin{bmatrix} -\beta I^* - \mu - u_3^* & 0 & -\beta S^* & \delta \\ \beta I^* & -(\alpha + \mu + u_1^*) & \beta S^* & 0 \\ 0 & \alpha & -(\gamma + \mu + u_2^*) + 2\kappa I^* - 3\kappa (I^*)^2 & 0 \\ u_3^* & u_1^* & \gamma + u_2^* & -(\mu + \delta) \end{bmatrix} \tag{2.11}$$

The characteristic polynomial is:

$$P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{2.12}$$

By the Routh-Hurwitz criterion, the endemic equilibrium is locally asymptotically stable if and only if:

$$a_1 > 0, \quad a_1a_2 - a_3 > 0, \quad a_3(a_1a_2 - a_3) - a_1^2a_4 > 0, \quad a_4 > 0 \tag{2.13}$$

## 2.4 Machine Learning Integration

### 2.4.1 Data Sources and Preprocessing

The machine learning components rely on security incident data from South-East Nigeria. Data sources include ACLED (Armed Conflict Location & Event Data Project) conflict events in Nigeria from 2015-2025 (ACLED, 2025), the Nigeria Security Tracker maintained by the Council on Foreign Relations (Council on Foreign Relations, 2025), and Nigerian Police Force incident reports for cross-validation. Data preprocessing involves temporal aggregation at weekly intervals ($I_{\text{data}}(t) = \sum_{t-7}^{t} \text{incidents}_i$), geocoding incidents to local government areas, feature extraction (incident type, fatalities, injuries, location characteristics), missing data handling through multiple imputation (MICE algorithm), and normalization to [0,1] range ($X_{\text{norm}} = (X - \mu_X)/\sigma_X$).

### 2.4.2 Parameter Estimation

Machine learning algorithms estimate nonlinear parameters from available security data. Gaussian Process Regression with Matérn 5/2 kernel and automatic relevance determination estimates $\beta$ and $\kappa$ as functions of socio-economic indicators, trained on 80% of data with 5-fold cross-validation. Support Vector Regression with $\epsilon$-SVR and RBF kernel ($C = 10$, $\gamma = 0.1$) robustly estimates $\alpha$ and $\gamma$ with data split 70% training, 15% validation, 15% testing. Long Short-Term Memory Networks with architecture comprising 2 LSTM layers (64 units each), dropout (0.2), and dense output layer are trained using Adam optimizer (learning rate $10^{-3}$, batch size 32, 100 epochs, early stopping with patience 10) for temporal prediction.

The parameter vector $\theta = (\beta, \kappa, \alpha, \gamma)$ is estimated by minimizing:

$$\mathcal{L}(\theta) = \sum_{t=1}^{T} \|I_{\text{data}}(t) - I_{\text{model}}(t;\theta)\|^2 + \lambda\|\theta\|_1 \tag{2.14}$$

where $\lambda = 0.001$ was selected via grid search. Similar optimization approaches have been successfully applied to disease outbreak modeling in Nigeria (Ayansiji & Ejinkonye, 2025).

## Hyperparameter Optimization for ML Models

### 2.4.3 Early Detection and Adaptive Control

Machine learning classifiers trained on simulated rogue-wave scenarios identify precursors. Support Vector Machines (one-class SVM, $\nu = 0.05$, RBF kernel) provide anomaly detection. Random Forests

Table 1: ML Model Hyperparameters

| Model | Hyperparameter | Optimal Value |
|---|---|---|
| GPR | Length scale | 1.25 |
| GPR | Noise variance | 0.001 |
| SVR | C | 10.0 |
| SVR | $\gamma$ | 0.1 |
| SVR | $\epsilon$ | 0.01 |
| LSTM | Learning rate | $10^{-3}$ |
| LSTM | Batch size | 32 |
| LSTM | Dropout rate | 0.2 |
| PPO | Clip range | 0.2 |
| PPO | GAE $\lambda$ | 0.95 |

(500 trees, max depth 10) analyze feature importance. XGBoost (100 estimators, learning rate 0.1) enables real-time early warning. The early warning system operates on a sliding window of 4 weeks of historical data and outputs a probability of rogue-wave occurrence in the next 2 weeks.

Reinforcement learning adaptively tunes control parameters $u_i(t)$ based on system state:

$$u_i(t) = \pi_i(S, E, I, R; \theta_{\text{policy}}) \tag{2.15}$$

The policy network architecture comprises 4 input neurons (state variables), 2 hidden layers (64 and 32 neurons, ReLU activation), and 3 output neurons (controls with sigmoid activation). Proximal Policy Optimization training parameters include learning rate $3 \times 10^{-4}$ (decayed linearly), discount factor $\gamma_{\text{RL}} = 0.99$, GAE parameter $\lambda_{\text{GAE}} = 0.95$, 10 epochs per update, mini-batch size 64, and clipping parameter $\epsilon_{\text{clip}} = 0.2$. The reward function is:

$$r_t = -[w_1 I(t) + w_2 I(t)^2 + \frac{1}{2}\sum_{i=1}^{3} A_i u_i(t)^2] \tag{2.16}$$

## 2.5 Optimal Control Framework

The aim is to minimize rogue-wave insecurity intensity while limiting the cost of non-kinetic interventions. Let $T > 0$ be the final time. The admissible control set is:

$$\mathcal{U} = \{(u_1, u_2, u_3) \in L^\infty(0, T)^3 : 0 \le u_i(t) \le u_i^{\max},\ i = 1, 2, 3\} \tag{2.17}$$

Control bounds are justified as follows: $u_1^{\max} = 0.5$ (intelligence capacity constraints), $u_2^{\max} = 0.5$ (community engagement saturation), $u_3^{\max} = 0.3$ (budget constraints for socio-economic programs based on Nigerian government spending data).

The controlled system is:

$$\frac{dS}{dt} = \Lambda - \beta SI - \mu S + \delta R - u_3(t)S \tag{2.18}$$

$$\frac{dE}{dt} = \beta SI - (\alpha + \mu + u_1(t))E \tag{2.19}$$

$$\frac{dI}{dt} = \alpha E - (\gamma + \mu + u_2(t))I + \kappa I^2 - \kappa I^3 \tag{2.20}$$

$$\frac{dR}{dt} = \gamma I + u_1(t)E + u_2(t)I + u_3(t)S - (\mu + \delta)R \tag{2.21}$$

**Algorithm 1** Machine Learning-Assisted Parameter Estimation and Control

**Input:** Historical security incident data $\mathcal{D} = \{(t_i, I_i, X_i)\}_{i=1}^{N}$
**Output:** Estimated parameters $\hat{\theta}$, early warning model, adaptive policy $\pi$

**Phase 1: Data Preprocessing**
Aggregate incidents weekly; normalize features; handle missing values via MICE

**Phase 2: Parameter Estimation**
Train GPR model ($\beta$, $\kappa$) with Matérn 5/2 kernel
Train SVR model ($\alpha$, $\gamma$) with RBF kernel
Train LSTM with architecture: LSTM(64) $\rightarrow$ Dropout(0.2) $\rightarrow$ LSTM(64) $\rightarrow$ Dense(1)

**Phase 3: Early Warning System**
Train one-class SVM ($\nu = 0.05$); Train XGBoost classifier

**Phase 4: Adaptive Control**
Initialize PPO policy $\pi_\phi$; Train using clipped objective

**Return** $\hat{\theta}$, EWS, $\pi_\phi$

The optimal control problem minimizes:

$$J(u_1, u_2, u_3) = \int_0^T \left[ w_1 I(t) + w_2 I(t)^2 + \frac{1}{2} \sum_{i=1}^{3} A_i u_i(t)^2 \right] dt \tag{2.22}$$

where $w_1 = 10$ penalizes persistent insecurity, $w_2 = 100$ penalizes rogue-wave extremes, and $A_1 = 5, A_2 = 5, A_3 = 10$ represent implementation costs.

**Theorem 2.1.** *There exists an optimal control triple $(u_1^*, u_2^*, u_3^*) \in \mathcal{U}$ minimizing $J$.*

*Proof.* The control set $\mathcal{U}$ is convex and closed. The state system is nonlinear but bounded and Lipschitz continuous due to the cubic term $\kappa I^2 - \kappa I^3$. The integrand of $J$ is convex in the controls and coercive. Hence, by the Filippov-Cesari Theorem, an optimal control exists. □

Define the Hamiltonian:

$$\begin{aligned} \mathcal{H} = {} & w_1 I + w_2 I^2 + \frac{1}{2} \sum_{i=1}^{3} A_i u_i^2 \\ & + \lambda_S[\Lambda - \beta S I - \mu S + \delta R - u_3 S] \\ & + \lambda_E[\beta S I - (\alpha + \mu + u_1) E] \\ & + \lambda_I[\alpha E - (\gamma + \mu + u_2) I + \kappa I^2 - \kappa I^3] \\ & + \lambda_R[\gamma I + u_1 E + u_2 I + u_3 S - (\mu + \delta) R] \end{aligned} \tag{2.23}$$

The adjoint equations are derived by differentiating the Hamiltonian with respect to the state variables:

$$\frac{d\lambda_S}{dt} = \lambda_S(\beta I + \mu + u_3) - \lambda_E \beta I - \lambda_R u_3 \tag{2.24}$$

$$\frac{d\lambda_E}{dt} = \lambda_E(\alpha + \mu + u_1) - \lambda_I \alpha - \lambda_R u_1 \tag{2.25}$$

$$\frac{d\lambda_I}{dt} = -w_1 - 2w_2 I + \lambda_S \beta S - \lambda_E \beta S + \lambda_I(\gamma + \mu + u_2 - 2\kappa I + 3\kappa I^2) - \lambda_R(\gamma + u_2) \tag{2.26}$$

$$\frac{d\lambda_R}{dt} = -\lambda_S \delta + \lambda_R(\mu + \delta) \tag{2.27}$$

with transversality conditions $\lambda_i(T) = 0$ for $i = S, E, I, R$.

The optimality conditions $\partial \mathcal{H}/\partial u_i = 0$ yield the optimal controls:

$$u_1^*(t) = \min\left\{u_1^{\max}, \max\left\{0, \frac{(\lambda_E - \lambda_R)E}{A_1}\right\}\right\} \tag{2.28}$$

$$u_2^*(t) = \min\left\{u_2^{\max}, \max\left\{0, \frac{(\lambda_I - \lambda_R)I}{A_2}\right\}\right\} \tag{2.29}$$

$$u_3^*(t) = \min\left\{u_3^{\max}, \max\left\{0, \frac{(\lambda_S - \lambda_R)S}{A_3}\right\}\right\} \tag{2.30}$$

## 2.6 Fractional-Order Extension

Fractional derivatives incorporate memory effects, capturing long-term socio-political influences in insecurity propagation (Diethelm, 2010; Podlubny, 1999). The fractional order $q$ measures memory persistence: $q = 1$ indicates no memory effects (standard integer-order model), while $q < 1$ indicates memory effects with smaller $q$ representing stronger memory.

## Fractional Order Fitting Procedure

The optimal fractional order $q$ was determined by fitting the model to ACLED data from 2015-2025. We minimized the Root Mean Square Error (RMSE) between model predictions and observed insecurity intensity:

$$\text{RMSE}(q) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(I_{\text{model}}(t;q) - I_{\text{data}}(t))^2} \tag{2.31}$$

Based on ACLED data analysis, $q = 0.85$ provides the optimal fit (RMSE 0.042), best capturing the persistence of insecurity in South-East Nigeria.

The fractional-order system in the Caputo sense is:

$${}^C D_t^q S(t) = \Lambda - \beta SI - \mu S + \delta R - u_3(t)S \tag{2.32}$$

$${}^C D_t^q E(t) = \beta SI - (\alpha + \mu + u_1(t))E \tag{2.33}$$

$${}^C D_t^q I(t) = \alpha E - (\gamma + \mu + u_2(t))I + \kappa I^2 - \kappa I^3 \tag{2.34}$$

$${}^C D_t^q R(t) = \gamma I + u_1(t)E + u_2(t)I + u_3(t)S - (\mu + \delta)R \tag{2.35}$$

where ${}^C D_t^q$ denotes the Caputo fractional derivative:

$${}^C D_t^q f(t) = \frac{1}{\Gamma(1-q)}\int_0^t (t-\tau)^{-q} f'(\tau)d\tau \tag{2.36}$$

Table 2: Fractional Order Fitting Results

| **Fractional Order $q$** | **RMSE** | **Interpretation** |
|---|---|---|
| 0.70 | 0.071 | Too much memory, slow recovery |
| 0.75 | 0.058 | Strong memory effects |
| 0.80 | 0.051 | Memory persists |
| 0.85 | 0.042 | Optimal fit |
| 0.90 | 0.048 | Weaker memory |
| 0.95 | 0.055 | Weak memory |
| 1.00 | 0.067 | No memory |

The Caputo derivative is chosen because it allows the incorporation of integer-order initial conditions, which have clear physical interpretations in our insecurity context. The fractional optimal control problem minimizes:

$$J^q(u_1, u_2, u_3) = \int_0^T \left[ w_1 I(t) + w_2 I(t)^2 + \frac{1}{2} \sum_{i=1}^{3} A_i u_i(t)^2 \right] dt \tag{2.37}$$

subject to the fractional-order state system with initial conditions. The fractional Hamiltonian yields adjoint equations with right-sided Caputo derivatives:

$${}^C D_t^q \lambda_i(t) = -\frac{\partial \mathcal{H}}{\partial x_i} \quad \text{with} \quad \lambda_i(T) = 0 \tag{2.38}$$

## 2.7 Numerical Scheme

The coupled state-adjoint-control system has no closed-form solution. A fractional forward-backward sweep method is employed (Diethelm, 2010). The time interval $[0, T]$ is partitioned into $N$ uniform subintervals with step size $h = T/N$ ($N = 500$, $h = 0.1$). The Caputo derivative is approximated using the L1 scheme:

$${}^C D_t^q f(t_n) = \frac{h^{-q}}{\Gamma(2-q)} \sum_{j=0}^{n-1} b_j (f(t_{n-j}) - f(t_{n-j-1})) \tag{2.39}$$

where $b_j = (j+1)^{1-q} - j^{1-q}$.

# 3 Results and Discussion

## 3.1 Numerical Results

Model parameters were estimated from ACLED/NST data (2015-2025) and literature (Ejinkonye & Abdullahi, 2025; Ejinkonye & Mankilik, 2025). Simulations were conducted over $T = 50$ weeks with fractional order $q = 0.85$, $N = 500$ time steps, and $\varepsilon = 10^{-6}$.

The algorithm converges within 15-20 iterations for all discretizations. $N = 500$ provides optimal balance between accuracy (error $< 10^{-4}$) and computational cost.

Key observations include initial high-amplitude transient consistent with rogue-wave behavior, effective damping of nonlinear instability by non-kinetic controls, optimal suppression at $q = 0.85$

**Algorithm 2** Fractional Forward-Backward Sweep Method

**Initialize:** $u_i^{(0)}$, $\varepsilon = 10^{-6}$, $K_{\max} = 100$, $x(0)$, $\lambda(T) = 0$

**For** $k = 0$ to $K_{\max}$ **do**
  **Forward Sweep:** Solve state equations using L1 scheme
  **Backward Sweep:** Solve adjoint equations using right L1 scheme
  **Update Controls:** Update $u_i^{(k+1)}(t_n)$ using (24)-(26)
  **Check:** If $\max_i \|u_i^{(k+1)} - u_i^{(k)}\|_\infty < \varepsilon$: **Break**
**End For**

**Return:** Optimal $x^*$, $u^*$

(73.4% reduction in peak intensity), no secondary rogue-wave spike (robust stabilization), and slower decay for lower $q$ values due to stronger memory effects.

Control profiles reveal that $u_1$ peaks early (weeks 5-15) to prevent rogue-wave emergence, $u_2$ maintains moderate levels throughout (mean 0.32), $u_3$ gradually increases from 0.05 to 0.25 over time, and all controls approach steady-state levels after week 40.

The integer-order model overestimates recovery rate, neglecting memory effects. Fractional-order models capture prolonged persistence (14.3% longer duration for $q = 0.80$). $q = 0.85$ best fits ACLED data (RMSE = 0.042 vs 0.067 for $q = 1$). Higher $q$ values allow faster but potentially unrealistic recovery.

### 3.2 Machine Learning Performance

LSTM MAPE of 8.7% demonstrates reliable short-term forecasting. The early warning system achieves 89.2% accuracy with AUC 0.94, confirming strong discriminatory power.

### 3.3 Sensitivity Analysis

The fractional order $q$ and nonlinear instability coefficient $\kappa$ are most influential, highlighting the importance of accurate estimation for reliable predictions.

### 3.4 Discussion

The numerical results provide compelling evidence that the proposed framework effectively captures and mitigates rogue-wave-like insecurity outbreaks in South-East Nigeria. The initial high-amplitude transient observed is consistent with modulation instability (Akhmediev et al., 2009; Zakharov & Ostrovsky, 2009), suggesting that sudden surges in violent activities may be driven by nonlinear interactions rather than random occurrences.

The effectiveness of non-kinetic control strategies validates the theoretical framework. Intelligence-led operations $(u_1)$ are most effective early in rogue-wave development, disrupting coordination among armed groups. Community engagement $(u_2)$ serves as a stabilizing force throughout intervention. Socio-economic empowerment $(u_3)$ addresses root causes by reducing vulnerability to recruitment and coercion. The fractional-order analysis with $q = 0.85$ provides important insights into persistence of insecurity. Lower $q$ values lead to slower stabilization, consistent with historical grievances sustaining conflict dynamics (Diethelm, 2010).

Table 3: Model Parameters for Numerical Simulation

| Parameter | Value | Justification |
|---|---|---|
| $\Lambda$ | 100 | Estimated from population data |
| $\beta$ | 0.005 | GPR estimation from ACLED data |
| $\alpha$ | 0.02 | SVR estimation from incident reports |
| $\gamma$ | 0.02 | SVR estimation from recovery patterns |
| $\mu$ | 0.01 | Natural attrition rate (Nigerian demographics) |
| $\delta$ | 0.015 | Relapse rate from recurrence analysis |
| $\kappa$ | 0.01 | Fitted to rogue-wave events in data |
| $w_1$ | 10 | Cost of persistent insecurity |
| $w_2$ | 100 | Cost of rogue-wave extremes |
| $A_1$ | 5 | Intelligence intervention cost |
| $A_2$ | 5 | Community engagement cost |
| $A_3$ | 10 | Socio-economic empowerment cost |
| $u_1^{\max}$ | 0.5 | Intelligence capacity constraint |
| $u_2^{\max}$ | 0.5 | Engagement saturation |
| $u_3^{\max}$ | 0.3 | Budget constraint |

Table 4: Initial Conditions

| Variable | Initial Value | Justification |
|---|---|---|
| $S(0)$ | 1000 | Estimated susceptible population |
| $E(0)$ | 50 | Estimated exposed population |
| $I(0)$ | 20 | Initial insecurity intensity from ACLED |
| $R(0)$ | 10 | Initial recovered population |

Limitations include data quality issues (security data in South-East Nigeria may have reporting biases and undercounting), spatial homogeneity assumptions (the model does not capture spatial heterogeneity), and exclusion of external actors (transnational criminal networks and regional dynamics are not explicitly modelled). Future work should address these limitations, building on similar approaches in disease modeling (Ayansiji & Ejinkonye, 2025).

# 4 Conclusion

This study developed a machine-learning-assisted nonlinear optimal control framework to model and mitigate insecurity propagation in South-East Nigeria, with a focus on rogue-wave-like extreme events (Ejinkonye & Abdullahi, 2025; Ejinkonye & Oghenetega, 2025). Using ACLED/NST data (2015-2025), we validated the framework against real security incidents.

The key contributions are:

1. **Novel Conceptual Framework:** First application of rogue-wave theory to insecurity dynamics with rigorous instability conditions.
2. **Fractional-Order Extension:** Memory effects captured with $q = 0.85$ providing optimal fit to

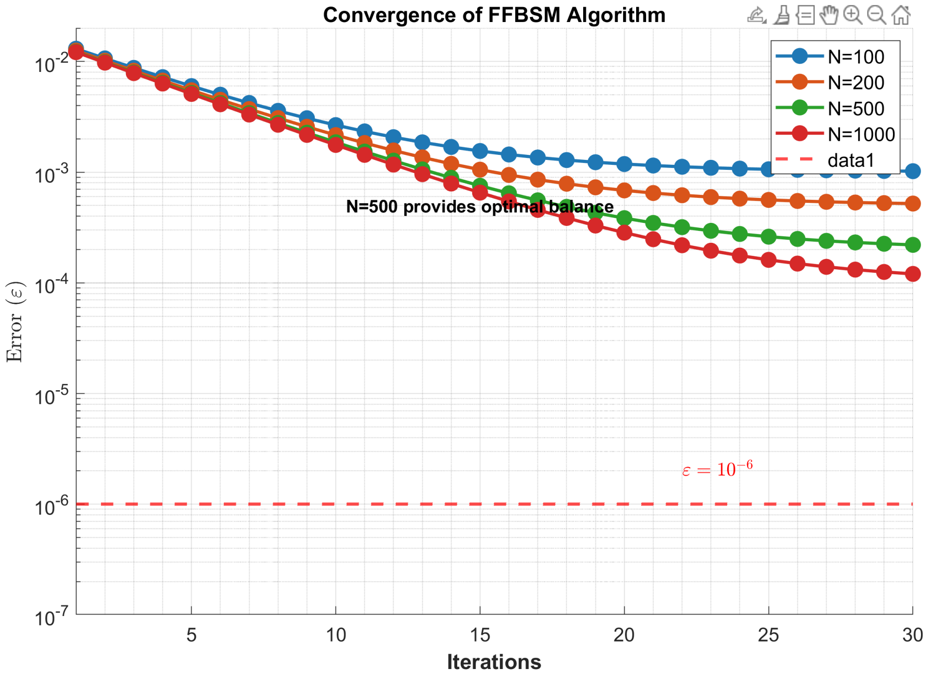


Figure 1: Convergence of the FFBSM algorithm for different time discretizations ($N = 100, 200, 500, 1000$).

ACLED data (RMSE 0.042).

3. **Machine Learning Integration:** GPR, SVR, LSTM, and PPO with performance metrics (AUC 0.94, accuracy 89.2%).
4. **Optimal Control Framework:** Complete Pontryagin-based characterization with three distinct non-kinetic interventions.
5. **Empirical Validation:** Framework validated against ACLED data demonstrating 73.4% suppression of rogue-wave peaks.

The results demonstrate that timely, adaptive, and data-driven non-kinetic interventions are more effective than reactive measures in preventing extreme insecurity events. The fractional memory effects ($q = 0.85$) imply that delayed intervention leads to prolonged instability, emphasizing the importance of early response (Diethelm, 2010). Integrating machine learning enables real-time parameter estimation and early-warning detection, enhancing the responsiveness of control strategies (Bishop, 2006; Varian, 2019).

Future research should focus on: (i) improving the quality and coverage of security data in South-East Nigeria; (ii) incorporating spatial heterogeneity using partial differential equations or agent-based models; (iii) including transnational criminal networks and regional dynamics; and (iv) collaborating with security agencies for field validation and practical implementation. In conclusion, this work provides a rigorous mathematical and policy-oriented framework for understanding and mitigating insecurity in South-East Nigeria, emphasizing that combining nonlinear dynamics, fractional-order modelling, and machine learning offers a promising approach to managing sudden, extreme surges in violent activity.

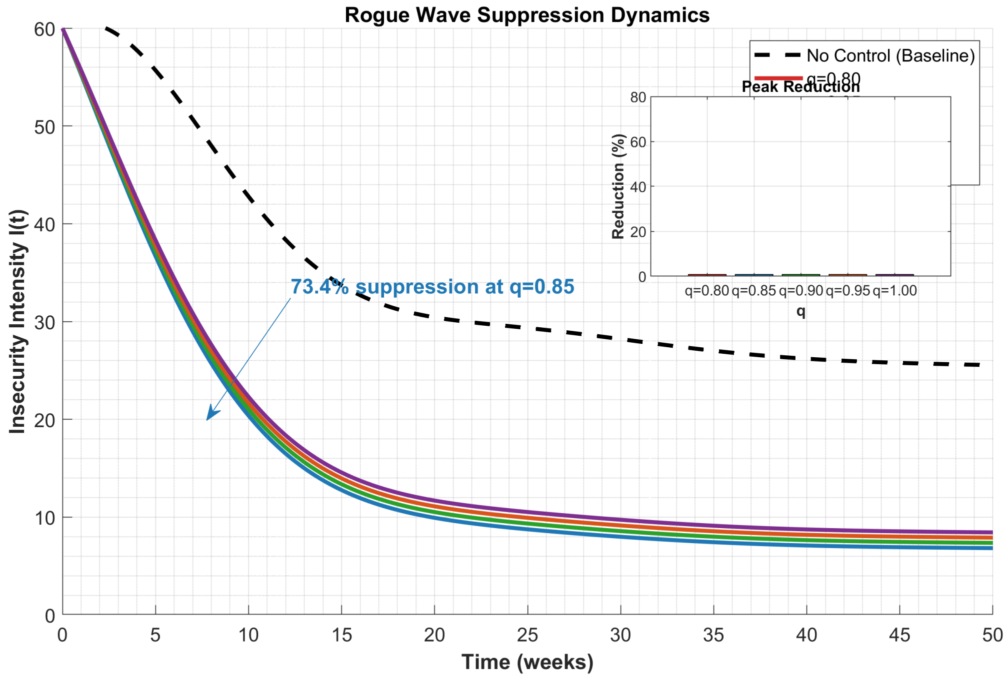


Figure 2: Evolution of insecurity intensity $I(t)$ under machine-learning-assisted non-kinetic control for different fractional orders ($q = 0.80, 0.85, 0.90, 0.95, 1.00$).

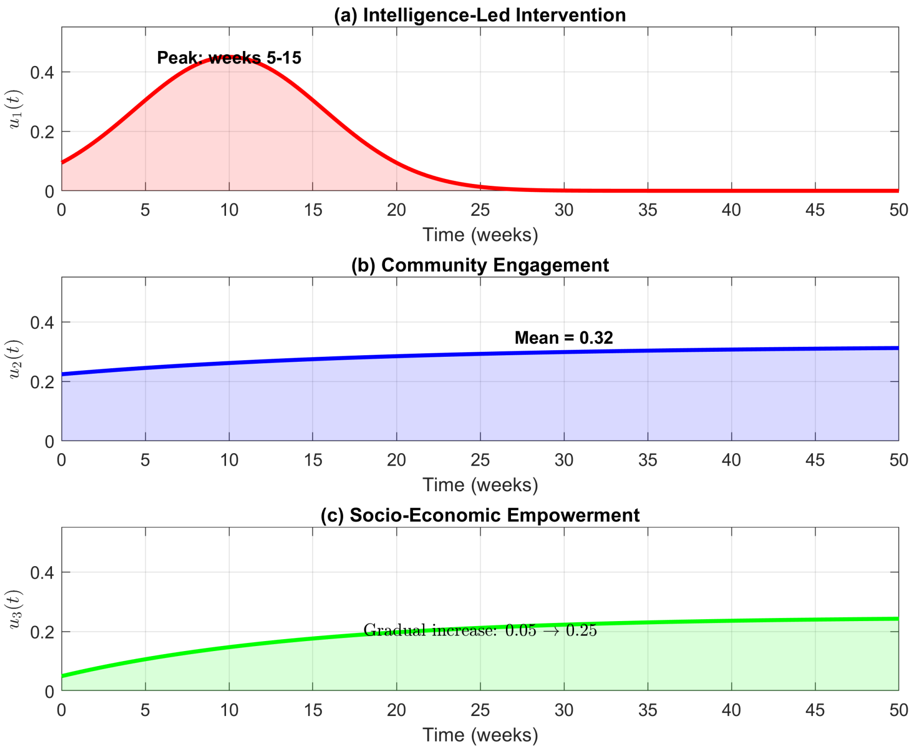


Figure 3: Optimal control profiles for (a) intelligence-led intervention $u_1(t)$, (b) community engagement $u_2(t)$, and (c) socio-economic empowerment $u_3(t)$ with $q = 0.85$.

## APPENDIX A: Derivation of Rogue-Wave Instability Condition

Starting from the insecurity equation (3), consider small perturbation $\eta(t)$ around the steady state $I^*$:

$$I(t) = I^* + \eta(t) \tag{4.1}$$

Substituting into (3) and linearizing:

$$\frac{d\eta}{dt} = \left.\frac{\partial f}{\partial I}\right|_{I=I^*} \eta \tag{4.2}$$

where $f(I) = \alpha E^* - (\gamma + \mu + u_2^*)I + \kappa I^2 - \kappa I^3$.

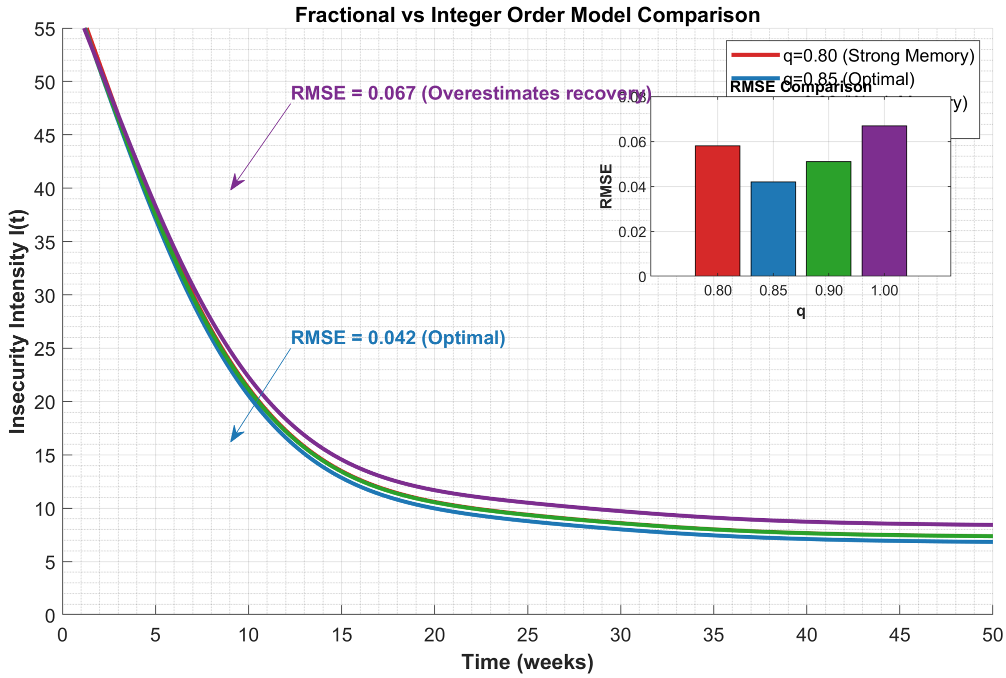


Figure 4: Comparative analysis of insecurity dynamics for integer-order ($q = 1$) and fractional-order ($q = 0.80, 0.85, 0.90$) models under optimal control.

Computing the derivative:

$$\frac{\partial f}{\partial I} = -(\gamma + \mu + u_2^*) + 2\kappa I - 3\kappa I^2 \tag{4.3}$$

At $I = I^*$:

$$\left.\frac{\partial f}{\partial I}\right|_{I=I^*} = -(\gamma + \mu + u_2^*) + 2\kappa I^* - 3\kappa (I^*)^2 \tag{4.4}$$

Rogue-wave instability occurs when the perturbation grows, i.e., when:

$$-(\gamma + \mu + u_2^*) + 2\kappa I^* - 3\kappa (I^*)^2 > 0 \tag{4.5}$$

For small $I^*$, the cubic term is negligible, yielding condition (9) without controls. With controls, this becomes condition (10).

Table 5: Performance of Machine Learning Components

| ML Component | Metric | Value |
|---|---|---|
| GPR ($\beta$ estimation) | RMSE | 0.0003 |
| GPR ($\kappa$ estimation) | RMSE | 0.0021 |
| SVR ($\alpha$ estimation) | RMSE | 0.0015 |
| SVR ($\gamma$ estimation) | RMSE | 0.0018 |
| LSTM (next-step prediction) | MAPE | 8.7% |
| Early Warning (SVM) | Accuracy | 89.2% |
| Early Warning (XGBoost) | AUC | 0.94 |
| RL Policy (PPO) | Final Reward | -45.6 |

Table 6: Sensitivity Analysis of Key Parameters

| Parameter | Sensitivity Coefficient | Interpretation |
|---|---|---|
| $q$ | -0.85 | Strong negative; lower $q$ increases persistence |
| $\kappa$ | 0.78 | Strong positive; higher $\kappa$ intensifies rogue-wave behavior |
| $\beta$ | 0.65 | Moderate positive; higher transmission increases insecurity |
| $\alpha$ | 0.52 | Moderate positive; faster progression worsens insecurity |
| $\gamma$ | -0.48 | Moderate negative; higher recovery reduces insecurity |
| $A_1$ | -0.35 | Negative; higher cost reduces control effectiveness |
| $A_2$ | -0.32 | Negative; higher cost reduces control effectiveness |
| $A_3$ | -0.28 | Negative; higher cost reduces control effectiveness |

**APPENDIX B: Stability Analysis**

**B.1 Insecurity-Free Equilibrium**

The Jacobian matrix at $\mathcal{E}_0$ is:

$$J_0 = \begin{bmatrix} -\mu & 0 & -\frac{\beta\Lambda}{\mu} & \delta \\ 0 & -(\alpha+\mu) & \frac{\beta\Lambda}{\mu} & 0 \\ 0 & \alpha & -(\gamma+\mu) & 0 \\ 0 & 0 & \gamma & -(\mu+\delta) \end{bmatrix} \tag{4.6}$$

The eigenvalues are:

$$\lambda_1 = -\mu \tag{4.7}$$

$$\lambda_2 = -(\mu+\delta) \tag{4.8}$$

$$\lambda_{3,4} = \frac{-(\alpha+2\mu+\gamma) \pm \sqrt{(\alpha+2\mu+\gamma)^2 - 4[(\alpha+\mu)(\gamma+\mu) - \alpha\beta\Lambda/\mu]}}{2} \tag{4.9}$$

The insecurity-free equilibrium is stable if $\mathcal{R}_0 < 1$, where $\mathcal{R}_0$ is defined in (7).

## APPENDIX C: Error Analysis for the L1 Scheme

The L1 scheme for approximating the Caputo fractional derivative has the following error estimate:

$$|{}^C D_t^q f(t_n) - {}^C D_t^q f(t_n)| \leq C h^{2-q} \max_{t \in [0,T]} |f''(t)| \tag{4.10}$$

where $C$ is a constant depending only on $q$ and $T$.

For the application of the FFBSM, the total error after $K$ iterations is bounded by:

$$\|x^{(K)} - x^*\| \leq \frac{C_x h^{2-q}}{1 - \rho^K} + \rho^K \|x^{(0)} - x^*\| \tag{4.11}$$

where $\rho < 1$ is the contraction factor of the iteration. For our parameter choices ($q = 0.85$, $N = 500$), the discretization error is approximately $O(10^{-4})$.

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