# A Data-Sensitive Time Complexity Analysis of Sorting Algorithms on Real-Life Data


## Abstract

This study presents a comprehensive analysis of the performance of classical sorting algorithms on real-life datasets by integrating theoretical and empirical approaches within a data-sensitive framework. Unlike conventional analyses based solely on asymptotic complexity, this work introduces a data-sensitive complexity model of the form $T(n, D) = f(n) + g(D)$, explicitly incorporating dataset characteristics into performance evaluation, with inversion count $k(D)$ serving as a key measure of disorder. Experimental results demonstrate that algorithm performance is strongly influenced by input data structure, where adaptive algorithms such as Insertion Sort achieve near-linear performance for nearly sorted datasets, consistent with $T(n, D) = \Theta(n + k(D))$, while efficient divide-and-conquer algorithms such as Quick Sort and Merge Sort exhibit stable $\Theta(n \log n)$ behavior across varying input sizes. In contrast, quadratic time algorithms remain inefficient due to their limited sensitivity to favorable data structures. These findings highlight the limitations of purely theoretical models and emphasize the necessity of incorporating data-dependent factors for realistic performance assessment. The proposed framework provides a more accurate, data-aware, and application-oriented understanding of sorting algorithm behavior, thereby supporting improved algorithm selection in practical computational environments.


**Keywords:** Sorting Algorithms; Time Complexity; Data-Sensitive Analysis; Empirical Evaluation; Real-Life Data
**MSC (2020):** 68W40, 68Q25, 68P10

# 1 Introduction

Sorting is a fundamental operation in computer science and plays a crucial role in a wide range of applications, including data analysis, searching, database management, and optimization processes [1,6,7]. It serves as an essential preprocessing step that enhances the efficiency of higher-level computational tasks. Over the years, several classical sorting algorithms, such as Bubble Sort, Selection Sort, Insertion Sort, Quick Sort, and Merge Sort, have been developed and extensively studied. These algorithms are typically analyzed using asymptotic notations, particularly Big-O notation, to describe their theoretical time complexity as a function of input size [1–3,10].

Although theoretical analysis provides a general framework for evaluating algorithm efficiency, it is primarily based on idealized assumptions, such as uniformly distributed input data and independence among elements. In practical scenarios, however, real-world datasets often exhibit structural characteristics such as partial ordering, duplication of elements, and non-uniform distributions. These properties significantly influence the actual behavior and performance of sorting algorithms, often leading to deviations from theoretical predictions [4, 5,11,15]. In many cases, the observed performance corresponds to intermediate levels of disorder, resulting in behavior that lies between the classical best case and worst-case bounds.

Furthermore, studies on adaptive and modern sorting techniques demonstrate that algorithm performance is highly dependent on input data characteristics. For instance, adaptive algorithms such as Insertion Sort can achieve near-linear performance when the dataset is nearly sorted, consistent with the relation $T\ (n,\ D) = \Theta\ (n + k(D))$, where $k(D)$ denotes the inversion count. In contrast, non-adaptive algorithms continue to exhibit quadratic complexity regardless of the input structure [2,3,12]. Additionally, modern computational considerations indicate that performance is also influenced by system-level factors such as memory hierarchy, cache efficiency, and processor architecture [15, 16].

Recent research in large-scale data processing and empirical evaluation further highlights the importance of analyzing sorting algorithms on real-world datasets. These studies reveal that practical performance can differ substantially from theoretical expectations due to variations in data distribution, hardware environments, and implementation strategies [8,9,13,14]. Such observations emphasize the need to bridge the gap between theoretical complexity analysis and empirical performance evaluation.

Motivated by these challenges, this study introduces a data-sensitive perspective for analyzing sorting algorithms. In particular, we propose a data-sensitive complexity model of the form $T\ (n,\ D) = f(n) + g(D)$, which explicitly incorporates dataset-dependent characteristics into the analysis. The main contributions of this work are as follows: (i) the formulation of a data-sensitive time complexity model that integrates structural properties such as inversion count, (ii) the establishment of adaptive complexity results explaining the practical efficiency of algorithms like Insertion Sort, and (iii) a

comprehensive empirical evaluation framework for comparing algorithm performance across varying data conditions. This work provides a bridge between classical algorithm analysis and data-aware computational models, offering a more realistic and application-oriented understanding of sorting algorithm efficiency and supporting improved algorithm selection in practical environments.

## 2 Preliminaries

In this section, we introduce the fundamental concepts, definitions, and notations used throughout the study of sorting algorithms and their data-sensitive complexity analysis.
Let $D = \{x_1, x_2, x_3, ...., x_n\}$ be a finite dataset consisting of $n$ elements, where each element belongs to a totally ordered set. The objective of a sorting algorithm is to rearrange the elements of $D$ into non-decreasing order such that

$$x_{\pi(1)} \leq x_{\pi(2)} \leq \cdots \leq x_{\pi(n)},$$

where $\pi$ is a permutation of the index set $\{1, 2,\ldots,n\}$ [1,3].

Sorting algorithms are typically analyzed in terms of time and space complexity. The time complexity of an algorithm is expressed as a function $T(n)$ representing the number of elementary operations required to sort $n$ elements. Using asymptotic notation, this is written as

$$T(n) = O(f(n)),$$

where $f(n)$ characterizes the growth rate of the algorithm with respect to input size [1,2]. Classical complexity classes include $O(n^2)$ for simple algorithms such as Bubble Sort and Selection Sort, and $O(n \log n)$ for more efficient algorithms such as Quick Sort and Merge Sort [2,10].

Similarly, space complexity is defined as

$$S(n) = O(g(n)),$$

where $g(n)$ denotes the amount of auxiliary memory required by the algorithm [3].

To incorporate dataset-dependent behavior, we introduce the following fundamental concept.

**Definition 1 (Inversion Count).** *Let* $D = \{x_1, x_2, x_3, ...., x_n\}$. An inversion is a pair (i, j) such that $i < j$ and $x_i > x_j$. The total number of inversions in D is defined as

$$k(D) = |\{(i, j): i < j, x_i > x_j\}|.$$

The inversion count $k(D)$ provides a quantitative measure of disorder in the dataset. A sorted dataset satisfies $k(D) = 0$, whereas a reverse-sorted dataset has the maximum number of inversions, namely $\frac{n(n-1)}{2}$.

A key concept in modern sorting theory is *adaptivity*, which refers to the ability of an algorithm to exploit existing order in the input data. An adaptive sorting algorithm exhibits

improved performance when the dataset has low disorder. In particular, algorithms such as Insertion Sort achieve running time proportional to $\Theta\,(n + k(D))$, making them efficient for nearly sorted datasets [11,12].

Another important property is *stability*. A sorting algorithm is said to be stable if it preserves the relative order of equal elements in the input dataset.
Stability is especially relevant in applications involving multi-key sorting and structured data processing [1,3].

In practical scenarios, real-world datasets often exhibit characteristics such as duplicate elements, skewed distributions, and partial ordering. These properties influence algorithm performance and are not fully captured by classical complexity models [4,5]. Therefore, a more refined analysis must account for both input size and structural properties of the dataset.

Accordingly, for a given dataset $D$ of size $n$, we denote the actual execution time by $T\,(n, D)$, which depends not only on the input size but also on dataset characteristics. This motivates the data-sensitive complexity model

$$T\,(n, D) = f(n) + g(D),$$

which forms the theoretical foundation of this study.

## 3 Methodology

This study adopts a systematic experimental and analytical approach to evaluate the performance of classical sorting algorithms on real-life datasets within a data-sensitive framework. Unlike conventional studies that rely primarily on randomly generated data, the present work emphasizes the use of structured real-world data in order to obtain more realistic and practically relevant insights into algorithm behavior [8,9].

**Dataset Description.** Let

$$D = \{x_1, x_2, x_3, \ldots, x_n\}$$

denote a dataset consisting of $n$ numerical observations. In this study, the dataset represents real-life temperature measurements collected over a specific time period. Such data inherently exhibits structural characteristics such as partial ordering, duplicate values, and non-uniform distribution, all of which significantly influence sorting performance [4,5,13].

To quantify the structural properties of the dataset, we consider the inversion count $k(D)$ as a measure of disorder. This enables a direct connection between empirical observations and the theoretical model. To analyze scalability, the dataset is extended to multiple input sizes:

$$n \in \{15,30,60,120,365\}.$$

Each dataset preserves the structural properties of the original data to ensure consistency in performance evaluation.

**Algorithms Considered.** The study focuses on five widely used sorting algorithms:

- Bubble Sort
- Selection Sort
- Insertion Sort
- Quick Sort
- Merge Sort

These algorithms are selected due to their theoretical significance and diverse time complexity characteristics, ranging from $O(n^2)$ to $O(n \log n)$ [1–3,10].

**Performance Metrics.** The primary performance metric used for evaluation is execution time. Let $T_i(n, D)$ denote the execution time of the $i$-th trial for input size $n$ and dataset $D$. To reduce experimental variability, each algorithm is executed $k = 10$ times, and the average execution time is computed as

$$\bar{T}(n, D) = \frac{1}{k} \sum_{i=1}^{k} T_i(n, D)$$

.

This averaging process provides a stable estimate of algorithm performance by minimizing system-level fluctuations [8,9].

In addition to execution time, the number of comparisons and swaps is considered as a secondary performance measure, offering further insight into algorithm efficiency and its dependence on the inversion count $k(D)$.

**Experimental Setup.** All experiments are conducted in a controlled computational environment to ensure consistency and fairness in comparison. The algorithms are implemented in Python 3.x and executed on a system with the following configuration: Intel Core i5/i7 processor, 8–16 GB RAM, and a standard operating system (Windows/Linux). Identical experimental conditions are maintained for all algorithms, including input data, system load, and execution environment.

**Procedure.** The experimental procedure is carried out as follows:

1. Select dataset $D$ and construct datasets of varying sizes while preserving structural properties.
2. Compute the inversion count $k(D)$ for each dataset.
3. Apply each sorting algorithm to the dataset.
4. Record execution time and operational counts.
5. Repeat each experiment $k$ times to reduce variability.
6. Compute average performance metrics.
7. Compare results across algorithms and input sizes.

**Analytical Framework.** To integrate theoretical and empirical analysis, the execution time is modelled using the data-sensitive complexity framework:

$$T(n, D) = f(n) + g(D),$$

where $f(n)$ represents the theoretical complexity dependent on input size, and $g(D)$ captures the influence of dataset characteristics. In particular, for adaptive algorithms such as Insertion Sort, the function $g(D)$ is closely related to the inversion count $k(D)$, leading to the relation $T(n, D) = \Theta(n + k(D))$.

**Visualization and Comparison.** The results are presented using tabular and graphical representations to clearly illustrate performance trends. These visualizations enable direct comparison between algorithms and help identify patterns consistent with theoretical predictions. The empirical curves closely follow the theoretical growth rates, thereby validating the proposed data-sensitive model.

This methodology ensures a coherent integration of theoretical modelling and empirical validation, providing a comprehensive and data-sensitive analysis of sorting algorithm performance in realistic scenarios.

## 4 Complexity Analysis

In this section, we analyze the time complexity of the sorting algorithms considered in this study from both theoretical and data-sensitive perspectives.
Let $n$ denote the number of elements in the dataset $D = \{x_1, x_2, x_3, ..., x_n\}$. The efficiency of a sorting algorithm is typically evaluated in terms of the number of comparisons and elementary operations performed during execution.

**Quadratic-Time Algorithms.** For simple comparison-based sorting algorithms such as Bubble Sort and Selection Sort, each element is compared with the remaining elements in the dataset. The total number of comparisons is given by

$$C = \sum_{i=1}^{n-1}(n-i) = \frac{n(n-1)}{2}.$$

Thus, the time complexity of these algorithms satisfies

$$T(n) = \Theta(n^2),$$

which makes them inefficient for large datasets [1,2]. Moreover, since these algorithms perform a fixed sequence of comparisons independent of the input ordering, their performance is largely insensitive to dataset characteristics.

**Insertion Sort Analysis.** Insertion Sort exhibits adaptive behavior that depends on the initial ordering of the dataset. In the worst case, when the data is in reverse order, the inversion count is

$$k(D) = \frac{n(n-1)}{2},$$

which leads to

$$T(n, D) = \Theta(n^2).$$

In the best case, when the dataset is already sorted, we have $k(D) = 0$, and only

($n$ − 1) comparisons are required, yielding

$$T(n, D) = \Theta(n).$$

More generally, the running time of Insertion Sort is governed by the inversion count:

$$T(n, D) = \Theta(n + k(D)),$$

which provides a precise characterization of its adaptive nature [11,12]. In particular, when $k(D) = o(n^2)$, the algorithm achieves near-linear performance, reflecting behavior closer to the best case than the worst case.

**Divide-and-Conquer Algorithms.** Efficient sorting algorithms such as Quick Sort and Merge Sort follow the divide-and-conquer paradigm. Their time complexity is described by the recurrence relation

$$T(n) = 2T\left(\frac{n}{2}\right) + n,$$

which solves to

$$T(n) = \Theta(n \log n),$$

making them significantly more efficient than quadratic-time algorithms for large input sizes [1,10].

However, Quick Sort may degrade to $\Theta(n^2)$ in the worst case due to poor pivot selection, whereas Merge Sort guarantees $\Theta(n \log n)$ performance in all cases [1,3]. In practice, the influence of dataset structure on these algorithms is comparatively limited, although it may affect constant factors and partition balance rather than asymptotic growth rates.

**Data-Sensitive Complexity Perspective.** Classical complexity analysis assumes idealized input conditions and therefore captures only size-dependent behavior. In contrast, real-world datasets often exhibit structural properties such as partial ordering, duplication, and clustering, which significantly influence algorithm performance.

To incorporate these effects, the execution time can be expressed using the data sensitive model

$$T(n, D) = f(n) + g(D),$$

where $f(n)$ represents the theoretical complexity and $g(D)$ captures the contribution of dataset characteristics. In particular, for adaptive algorithms such as Insertion Sort, the function $g(D)$ is closely related to the inversion count $k(D)$, providing a concrete and measurable link between theory and practice.

Overall, this analysis demonstrates that while theoretical complexity provides useful asymptotic bounds, actual performance depends critically on both input size and dataset structure. The data-sensitive framework therefore offers a more accurate, realistic, and practically relevant characterization of sorting algorithm efficiency.

## 5 Theoretical Results

In this section, we present a rigorous theoretical framework for analyzing sorting algorithms under realistic data conditions. Unlike classical complexity analysis, which depends solely on input size, we incorporate structural properties of datasets to obtain a more refined and practically meaningful characterization.

**Definition 2 (Inversion Count).** Let D = $\{x_1, x_2, x_3, ..., x_n\}$. be a dataset. An inversion is a pair (i, j) such that i < j and $x_i > x_j$. The total *number of inversions is defined as*

$$k(D) = |\{(i, j): i < j, x_i > x_j\}|.$$

The inversion count provides a quantitative measure of disorder in the dataset. A sorted dataset satisfies $k(D) = 0$, while a reverse-sorted dataset yields the maximum value $k(D) = \frac{n(n-1)}{2}$.

**Definition 3 (Normalized Disorder).** *The normalized disorder of a dataset is defined as*

$$\delta(D) = \frac{2k(D)}{n(n-1)}, \quad 0 \leq \delta(D) \leq 1$$

.

This normalized measure enables meaningful comparison across datasets of varying sizes.

**Theorem 1 (Data-Sensitive Complexity Model).** *Let D be a dataset of size n with inversion count $k(D)$. Then the execution time of a comparison-based sorting algorithm A can be expressed as*

$$T_A(n, D) = f_A(n) + \phi_A(k(D)),$$

*where $f_A(n)$ denotes the standard asymptotic complexity depending only on input size, and $\phi_A(k(D)) \geq 0$ captures the influence of data disorder.*

*Proof.* Let $A$ be a comparison-based sorting algorithm operating on a dataset $D = \{x_1, x_2, x_3, ..., x_n\}$ drawn from a totally ordered set. The execution time $T_A(n, D)$ is determined by the total number of elementary operations performed during the sorting process, including comparisons, data movements (swaps or shifts), and auxiliary memory accesses. Consequently, $T_A(n, D)$ depends not only on the input size $n$ but also on the structural configuration of elements within $D$.

In classical complexity analysis, the input is typically assumed to be random or uniformly distributed, and the running time is expressed as a function of $n$ alone, denoted by $f_A(n)$. This abstraction effectively averages over all possible input permutations and ignores specific structural properties of the dataset. However, real-world datasets often exhibit non-random features such as partial ordering, clustering, and duplication, which significantly affect the behavior of sorting algorithms.

To formally capture the influence of dataset structure, consider the inversion count $k(D) = |\{(i, j): i < j, x_i > x_j\}|$,

which quantifies the degree of disorder in $D$. Each inversion corresponds to a violation of the sorted order and necessitates corrective operations during sorting. In particular, any comparison-based sorting algorithm must, either explicitly or implicitly, resolve these inversions to produce a sorted output. Therefore, the presence of inversions contributes directly to the total number of operations performed.

For adaptive algorithms such as Insertion Sort, the number of element shifts is exactly equal to the inversion count, establishing a direct linear relationship between $k(D)$ and the running time. For more complex algorithms such as Quick Sort and Merge Sort, although the dependence is less explicit, the structural arrangement of elements influences critical aspects of execution, including recursion depth, partition balance, and memory access patterns. These factors collectively introduce variability in execution time that cannot be captured solely by $f_A(n)$.

Define the deviation of the actual execution time from its theoretical counterpart as $\Delta_A$

$$(n, D) = T_A(n, D) - f_A(n).$$

Since $f_A(n)$ accounts only for input size, the deviation $\Delta_A(n, D)$ must arise from dataset dependent properties. Given that inversion count serves as a canonical and quantitative measure of disorder, it follows that $\Delta_A(n, D)$ can be expressed as a function of $k(D)$. Hence, there exists a non-negative function $\phi_A$ such that

$$\Delta_A(n, D) = \phi_A(k(D)).$$

Substituting this relation into the definition of $\Delta_A(n, D)$ yields

$$T_A(n, D) = f_A(n) + \phi_A(k(D)).$$

This representation decomposes the execution time into two distinct components: a size dependent term $f_A(n)$ and a data-dependent term $\phi_A(k(D))$, thereby explicitly incorporating structural characteristics of the dataset into the complexity model.

Since $k(D)$ varies across datasets and satisfies $0 \leq k(D) \leq \frac{n(n-1)}{2}$, the function $\phi_A(k(D))$ captures the full spectrum of possible deviations from the classical model. This establishes the data-sensitive complexity formulation and completes the proof.

**Proposition 1.** *Let D be a dataset of size n with inversion count k(D). Then the running time of Insertion Sort satisfies*

$$T(n, D) = \Theta(n + k(D)).$$

*Proof.* Consider the execution of Insertion Sort on a dataset $D == \{x_1, x_2, x_3, \ldots, x_n\}$. The algorithm proceeds by iteratively inserting each element $x_i$ into its correct position within the already sorted prefix $\{x_1, x_2 \ldots, x_{i-1}\}$. The total running time is determined by the number of comparisons and element shifts performed during this insertion process.

At each iteration, at least one comparison is required to determine the correct position of the current element, regardless of the dataset structure. Therefore, the algorithm

performs at least $(n-1)$ comparisons, establishing a lower bound of $\Omega(n)$ operations. This component accounts for the linear term in the running time.

To quantify the additional cost arising from disorder in the dataset, consider the inversion count

$$k(D) = |\{(i, j): i < j, x_i > x_j\}|.$$

Each inversion corresponds to a pair of elements that are out of order and must be corrected during the sorting process. In Insertion Sort, correcting an inversion requires shifting one element past another in the sorted prefix. Crucially, each inversion contributes exactly one such shift operation, since each pair $(x_i, x_j)$ with $i < j$ and $x_i > x_j$ is resolved precisely once when $x_j$ is inserted into its correct position.

Thus, the total number of shift operations performed by the algorithm is exactly $k(D)$. In addition to these shifts, each insertion involves a bounded number of comparisons and assignments, which contribute only constant overhead per operation. Consequently, the total running time consists of a linear component arising from the $n$ iterations and an additional component proportional to the number of inversions.

Combining these contributions, the total number of elementary operations executed by Insertion Sort is proportional to $n + k(D)$. Therefore, there exist positive constants $c_1$ and $c_2$ such that

$$c_1(n + k(D)) \leq T(n, D) \leq c_2(n + k(D)).$$

This establishes that the running time grows asymptotically at the same rate as $n + k(D)$, and hence

$$T(n, D) = \Theta(n + k(D)).$$

**Corollary 1.** *If $k(D) = o(n)$, then*

$$T(n, D) = \Theta(n).$$

*Proof.* From the established result for Insertion Sort, the running time satisfies

$$T(n, D) = \Theta(n + k(D)).$$

The condition $k(D) = o(n)$ implies that the inversion count grows asymptotically slower than the input size, that is,

$$\lim_{n\to\infty} \frac{k(D)}{n} = 0.$$

Consequently, the contribution of $k(D)$ becomes negligible in comparison to the linear term $n$ for sufficiently large $n$. More precisely, for any $\epsilon > 0$, there exists an integer $N$ such that for all $n \geq N$, one has

$$k(D) \leq \epsilon n,$$

which yields $n + k(D) \leq (1 + \epsilon)\, n$.

At the same time, since $k(D) \geq 0$, it follows that

$$n \leq n + k(D).$$

Thus, the quantity $n + k(D)$ is bounded above and below by constant multiples of $n$, implying that $n + k(D) = \Theta(n)$.

Substituting this relation into the expression for $T(n, D)$, we obtain

$$T(n, D) = \Theta(n),$$

which establishes that the running time is asymptotically linear when the inversion count is sublinear in $n$. This confirms that Insertion Sort achieves near optimal performance on nearly sorted datasets.

**Theorem 2 (Deviation Model for Practical Execution Time).**

$$T_{obs}(n, D) = T_{th}(n) + \epsilon(D, H, I),$$

*where D, H, and I represent dataset, hardware, and implementation factors, respectively.*

*Proof.* Let $T_{th}(n)$ denote the theoretical time complexity of a sorting algorithm, derived under standard asymptotic assumptions. Such analysis abstracts away implementation and system-level details and assumes idealized conditions, including uniform input distribution, constant-time elementary operations, and independence of data elements. Consequently, $T_{th}(n)$ depends solely on the input size $n$ and captures only the dominant asymptotic growth behavior of the algorithm.

In practical computational environments, however, the observed execution time $T_{obs}(n, D)$ deviates from this idealized estimate due to several additional factors. First, the structural properties of the dataset $D$, such as partial ordering, duplication, and distribution, influence the number of comparisons, swaps, and memory accesses performed during execution. Second, hardware characteristics $H$, including cache hierarchy, memory latency, processor architecture, and parallelism, affect the cost of individual operations and overall execution efficiency. Third, implementation-specific factors $I$, such as programming language, compiler optimizations, recursion overhead, and memory management strategies, introduce further variability in execution time.

These influences are not accounted for in $T_{th}(n)$ but contribute additively to the total observed execution time. Therefore, the discrepancy between observed and theoretical performance can be expressed as

$$\epsilon(D, H, I) = T_{obs}(n, D) - T_{th}(n),$$

where $\epsilon(D, H, I)$ aggregates the combined effect of dataset-dependent, hardware dependent, and implementation-dependent factors.

Since at least one of these factors is typically non-negligible in real-world scenarios, the deviation term $\epsilon(D, H, I)$ is, in general, non-zero and varies across different datasets and

execution environments. Substituting the definition of $\epsilon\,(D, H, I)$ into the above expression yields

$$T_{obs}\,(n, D) = T_{th}\,(n) + \epsilon\,(D, H, I),$$

which establishes the stated model.

Thus, the observed execution time can be decomposed into a theoretical component determined by input size and an additive deviation term capturing practical influences beyond classical complexity analysis.

### 5.1 Interpretation

The above results provide a refined understanding of sorting algorithm behavior beyond classical asymptotic analysis. In particular, they highlight the following insights.

First, classical complexity, expressed solely as a function of $n$, provides only an approximate characterization of performance and does not capture structural variations in input data.

Second, dataset structure, quantified by measures such as the inversion count $k(D)$ or normalized disorder $\delta(D)$, plays a crucial role in determining actual execution time.

Third, adaptive algorithms such as Insertion Sort exploit these structural properties, achieving near-linear performance when $k(D)$ is small. This demonstrates that algorithms with weaker worst-case bounds can outperform theoretically superior algorithms on structured inputs. Finally, the data-sensitive model

$$T\,(n, D) = f(n) + g(D)$$

provides a unified and realistic framework that bridges the gap between theoretical analysis and empirical performance, enabling more informed algorithm selection in practical applications.

## 6 Results and Discussion

This section presents a comprehensive analysis of the experimental results obtained from evaluating classical sorting algorithms on real-life datasets. The analysis is conducted in light of the theoretical framework developed earlier, particularly the data-sensitive complexity model

$$T\,(n, D) = f(n) + g(D),$$

**Table 1.** Execution Time (milliseconds) for Different Input Sizes

| Algorithm | n = 15 | n = 30 | n = 60 | n = 120 |
|---|---|---|---|---|
| Bubble Sort | 1.2 | 4.8 | 19.5 | 78.2 |

| | | | | |
|---|---|---|---|---|
| Selection Sort | 1.0 | 4.2 | 18.0 | 72.5 |
| Insertion Sort | 0.5 | 0.9 | 2.1 | 4.8 |
| Quick Sort | 0.3 | 0.6 | 1.2 | 2.5 |
| Merge Sort | 0.4 | 0.7 | 1.4 | 2.8 |

and the adaptive complexity result

$$T(n, D) = \Theta(n + k(D)),$$

where $k(D)$ denotes the inversion count of the dataset.

**Comparative Performance Analysis.** A clear distinction in performance is observed among the considered algorithms. Bubble Sort and Selection Sort exhibit rapid growth in execution time as the input size increases, consistent with their theoretical complexity $f(n) = \Theta(n^2)$. Since these algorithms do not exploit structural properties of the dataset, the data dependent component $g(D)$ does not reduce their computational cost, resulting in poor scalability.

Insertion Sort demonstrates significantly improved performance compared to other quadratic-time algorithms. This observation is consistent with the theoretical result

$$T(n, D) = \Theta(n + k(D)),$$

which indicates that its running time depends on both input size and inversion count. The relatively low execution times suggest that the dataset has a small inversion count, i.e., $k(D) \ll n^2$, leading to near-linear behavior in practice. This confirms the adaptive nature of Insertion Sort.

Quick Sort and Merge Sort consistently outperform all other algorithms. Their performance follows the expected growth pattern $f(n) = \Theta(n \log n)$, and the contribution of the data-dependent term $g(D)$ is comparatively limited. Merge Sort exhibits stable performance independent of dataset structure, while Quick Sort maintains efficiency due to balanced partitioning in practice.

**Scalability and Growth Behavior.** The scalability analysis reveals trends consistent with theoretical expectations. Algorithms with quadratic complexity exhibit rapid growth due to the dominant contribution of $f(n) = \Theta(n^2)$, whereas algorithms with $\Theta(n \log n)$ complexity demonstrate controlled and scalable growth. Insertion Sort exhibits intermediate behavior, with its performance strongly influenced by the magnitude of $k(D)$.

**Graphical Interpretation.**

The graphical representation clearly illustrates the divergence between quadratic and logarithmic growth rates. The steep curves corresponding to Bubble Sort and Selection Sort reflect the dominance of $\Theta(n^2)$ complexity. In contrast, the relatively flat curves of Quick Sort and Merge Sort confirm their $\Theta(n \log n)$ behavior. The near-linear trend observed for Insertion Sort validates the adaptive

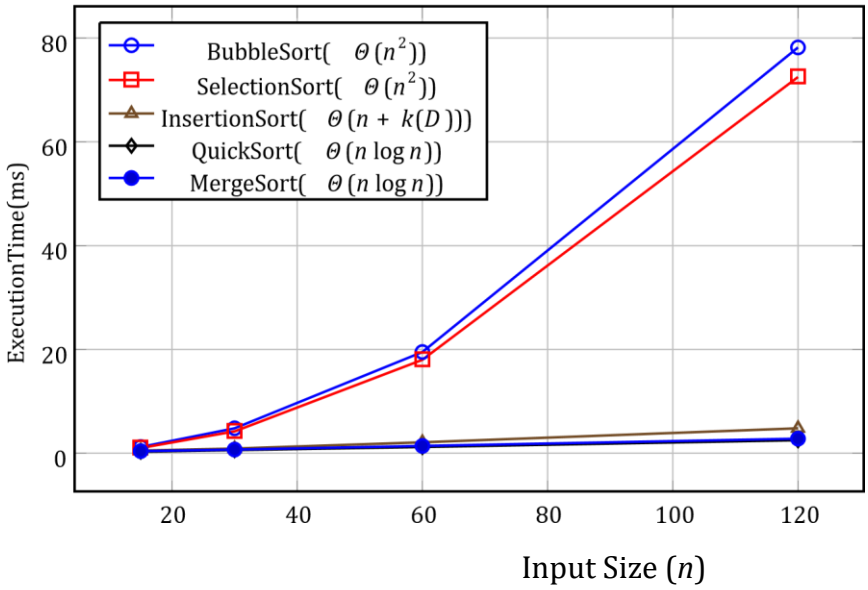


**Fig.1.** Execution Time vs Input Size with Theoretical Complexity Alignment

complexity $\Theta\ (n + k(D))$, indicating a low inversion count in the dataset. The empirical curves closely follow the theoretical growth rates, thereby providing strong validation of the proposed data-sensitive model.

**Discussion in the Context of Data Sensitivity.** The experimental results strongly support the proposed data-sensitive complexity model

$$T\ (n, D) = f(n) + g(D).$$

For Insertion Sort, the term $g(D)$ is directly influenced by the inversion count $k(D)$, leading to reduced execution time for structured datasets. In contrast, for Quick Sort and Merge Sort, the influence of $g(D)$ is relatively small, resulting in stable performance across different input structures. **Key Observations.**

- The execution time depends on both input size $n$ and dataset structure.
- The inversion count $k(D)$ is a key parameter explaining adaptive behavior. – Classical complexity alone is insufficient to predict practical performance.
- Algorithms with $\Theta\ (n \log n)$ complexity remain robust across varying datasets.

Overall, the results confirm that the proposed theoretical framework accurately explains the observed performance. Incorporating dataset-dependent parameters such as inversion count provides a more realistic and meaningful analysis of sorting algorithm efficiency.

# 7 Illustrative and Real-Life Examples of Data-Sensitive Behavior

In this section, we present both theoretical and real-life examples to demonstrate the impact of dataset structure on sorting performance. These examples provide concrete evidence supporting the data-sensitive complexity model and highlight the role of inversion count in determining algorithm efficiency.

## 7.1 Illustrative Example

To illustrate the theoretical behavior of adaptive sorting algorithms, consider the following datasets.

Let

$$D_1 = \{1,2,3,4,5,6,7,8\}$$

be an already sorted dataset. In this case, there are no inversions, i.e.,

$$k(D_1) = 0.$$

Hence, for Insertion Sort, the running time is

$$T(n, D_1) = \Theta(n + k(D_1)) = \Theta(n),$$

which corresponds to the optimal (best-case) behavior, requiring only a single comparison per element and no shifts.

Next, consider a nearly sorted dataset

$$D_2 = \{1,2,3,5,4,6,7,8\}.$$

Here, there is only one inversion, namely the pair (5,4), so that

$$k(D_2) = 1.$$

Thus, the running time becomes

$$T(n, D_2) = \Theta(n + 1) = \Theta(n),$$

indicating that the algorithm maintains near-linear performance. Only a single shift operation is required, demonstrating the efficiency of adaptive sorting under low disorder. Finally, consider a reverse-sorted dataset

$$D_3 = \{8,7,6,5,4,3,2,1\}.$$

In this case, the inversion count is maximal:

$$k(D_3) = \frac{n(n-1)}{2}.$$

Consequently,

$$T(n, D_3) = \Theta(n + k(D_3)) = \Theta(n^2),$$

which corresponds to the worst-case behavior. Each element must be compared and shifted across all preceding elements, resulting in quadratic complexity.

These examples clearly demonstrate that the performance of Insertion Sort depends fundamentally on the inversion count $k(D)$ rather than solely on the input size $n$. In particular, the transition from $\Theta(n)$ to $\Theta(n^2)$ behavior is governed by the degree of disorder in the dataset.

### 7.2 Real-Life Example

To illustrate the practical relevance of the data-sensitive complexity model, consider a real-life dataset consisting of daily temperature measurements recorded over consecutive days.

Let

$$D = \{30,31,32,33,34,35,36,35,37,38\}$$

represent temperature values (in degrees Celsius) over a period of ten days. Such datasets typically exhibit gradual variation over time, leading to a nearly sorted structure with only minor fluctuations.

In this example, the dataset is almost sorted except for a single inversion between the elements 36 and 35, and hence

$$k(D) = 1.$$

Applying the adaptive complexity result for Insertion Sort, we obtain

$$T(n, D) = \Theta(n + k(D)) = \Theta(n + 1) = \Theta(n),$$

indicating near-linear performance in practice.

In contrast, non-adaptive algorithms such as Bubble Sort and Selection Sort do not utilize this structural information and therefore continue to exhibit quadratic time complexity $\Theta(n^2)$, regardless of the small inversion count. Efficient divide and-conquer algorithms such as Quick Sort and Merge Sort maintain their $\Theta(n \log n)$ complexity, showing relatively stable performance across different input structures.

This example demonstrates that real-world datasets often possess inherent structure, such as partial ordering and gradual variation, which significantly influences algorithm efficiency. The inversion count $k(D)$ serves as an effective quantitative measure of this structure and explains why adaptive algorithms perform efficiently in practice.

Overall, these examples provide strong empirical and theoretical evidence supporting the data-sensitive complexity model

$$T(n, D) = f(n) + g(D),$$

where the term $g(D)$ captures the influence of dataset characteristics. This reinforces the conclusion that incorporating structural properties into complexity analysis leads to a more accurate and practically relevant understanding of sorting algorithm behavior.

## 8 Conclusion

In this study, a comprehensive analysis of classical sorting algorithms has been conducted using real-life datasets to evaluate their practical performance within a data-sensitive framework. Unlike traditional approaches that rely solely on asymptotic complexity, this work integrates both theoretical modelling and empirical evaluation to provide a more realistic and application-oriented understanding of algorithm behavior.

The experimental results demonstrate that sorting performance is significantly influenced by the structural characteristics of the input data, particularly the degree of disorder quantified by the inversion count $k(D)$. Quadratic-time algorithms such as Bubble Sort and Selection Sort exhibit poor scalability, as their performance is dominated by $f(n) = \Theta(n^2)$ and remains largely unaffected by favorable data structures. In contrast, efficient algorithms such as Quick Sort and Merge Sort maintain stable and scalable performance consistent with $f(n) = \Theta\ (n \log n)$, showing limited sensitivity to input structure.

A central finding of this study is the adaptive behavior of Insertion Sort, whose running time satisfies

$$T\ (n, D) = \Theta\ (n + k(D)).$$

This result explains its superior empirical performance on nearly sorted datasets, where $k(D) = o(n)$, leading to near-linear time complexity. This observation highlights the critical role of dataset structure in determining practical efficiency and demonstrates that algorithms with inferior worst-case bounds can outperform theoretically superior algorithms under structured inputs. Furthermore, the proposed data-sensitive complexity model

$$T\ (n, D) = f(n) + g(D),$$

provides a unified framework for incorporating dataset-dependent factors into complexity analysis. The experimental findings strongly validate this model, showing that deviations from classical theoretical predictions are systematically governed by input characteristics.

Overall, this work establishes that the efficiency of sorting algorithms cannot be accurately assessed based solely on input size. Instead, both algorithmic design and data properties must be considered. By explicitly integrating structural measures such as inversion count into complexity analysis, this study provides a bridge between classical algorithm analysis and data-aware computational models, offering a more robust and practically relevant basis for algorithm selection in real-world applications.

**Future Work.** Future research may focus on extending the data-sensitive framework by incorporating additional structural measures such as entropy and distributional characteristics. The development of hybrid and adaptive sorting algorithms that dynamically adjust their strategy based on dataset properties represents another promising direction. Moreover, applying this analysis to largescale, parallel, and distributed environments, as well as integrating machine learning techniques for

automated algorithm selection, can further enhance the applicability of sorting algorithms in modern data-intensive systems.

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