Fixed Pointsfor“O-W-C”- Self- Mappings on

C-M-Spaces

Abstract

In the presentresearch article, we obtained a unique common fixed points for “O-W-C”(Occasionally Weakly Compatible) self-mappings with satisfying a generalized contractive type condition on “C- M-Space”(Cone –Metric-Space). Our main aim toextends, improves and generalizes recent comparable existing theorems in the references.

Key Words:C-M-Space(Cone Metric Space), fixed point, common fixedpoint, fixed point theorem, and O-W-C(Occasionally Weakly Compatible).

AMS -2010 Subject Classification:“54H25” and “47H10”.

1. INTRODUCTION

Inthe Non- linear analysis, the study of fixed point results are very most important tool. And the concept of a cone metric space was introduced by the authors “Huang” and “Zhang” [6]. And they have been generalized a metric space concept , that is , cone metric space , for this they have been replaced the “real numbers” by an ordered “Banach” space and also got some of the common fixed point resultson “C-M-Space”(Cone –Metric-Space). Later on, many authorshas beengeneralized, improved and extended “Huang” and “Zhang” [6] results in so many forms(for e.x.,[1-5],[7-12]). And recent times “Bhatt” and “Chandra”[5]have gotsome fixed point results in “O-W-C”(Occasionally Weakly Compatible) self-mappingson C-M-Spaces(Cone –Metric-Spaces). Our main aim is in this research article we proved fixed point result for “O-W-C”(Occasionally Weakly Compatible) self-mappings on “C- M-Spaces”.

2. PRELIMINARIES

In ordered to get fixed point results, we need some of the basic Lemmas &Definitionswhich are in [6, 7].

2.1.Definition.Supposethat$S_1$is a real “Banach” space. And a subset T_1 of S_1 is said to be a cone iff

(A) $T_1$is not empty and is closed and $T_1 \neq \{0\}$;

(B) $\alpha_1, \beta_1 \in R$, $\alpha_1 ,\beta_1 \geq 0$, $u_1, v_1 \in T_1$implies $\alpha_1 u_1 + \beta_1 v_1 \in T_1$;

(Ci) $T_1 \cap (-T_1) = \{0\}$.

For a “cone”$T_1 \subset S_1$, defining a partial order “$\leq$” with respect to $T_1$by

“$\alpha_1 \leq \beta_1 \Leftrightarrow \beta_1 - \alpha_1 \in T_1$“. A cone T_1 is said to be“normal” if there is a number $M_1 > 0$such that for all $\alpha_1, \beta_1 \in T_1$,

“$0 \leq \alpha_1 \leq \beta_1 \Rightarrow \|\alpha_1\| \leq M_1 \|\beta_1\|$”.

The smallest +vee number satisfying the above inequality is called the “normal constant” of T_1, while“$\alpha_1 << \beta_1$“ stands for “$\beta_1 - \alpha_1$“ interior of T_1.

2.2.Definition. Supposethat$X_1$is a non-empty set of S_1. And suppose that the map

$\rho: X_1 \times X_1 \rightarrow S_1$ satisfying the following:

(A). $0 \leq \rho(\alpha_1, \beta_1)$ for all $\alpha_1, \beta_1 \in X_1$ and $\rho(\alpha_1, \beta_1) = 0$ if and only if $\alpha_1 = \beta_1$;
(B). $\rho(\alpha_1, \beta_1) = \rho(\beta_1, \alpha_1)$ for all $\alpha_1, \beta_1 \in X_1$;
(C). $\rho(\alpha_1, \beta_1) \leq \rho(\alpha_1, \gamma_1) + \rho(\gamma_1, \beta_1)$ for all $\alpha_1, \beta_1, \gamma_1 \in X_1$.

Then $\rho$is called a “C-Metric “(Cone -Metric) on $X_1$and (X_1, ρ) is said to be a “C-M-Space”(Cone –Metric-Space).

2.3.Definition. Let (X_1, ρ) be a C-M-Space .We said that $\{x_n\}$ is a
(A). Cauchy sequence if for every $b_1 \epsilon_1$ with “$b_1 >> 0$”, then , $\exists$ a natural number $n_1 \ni$ $\rho(x_n, x_m) << b_1$, for all $n, m > n_1$.
(B). convergent sequence if for every “$b_1 \epsilon S_1$“ with“$b_1 >> 0$”, then $\exists$ a natural number $N_1 \ni$ $\rho(x_n, x_1) << b_1$, for all $n > n_1$for some fixed x_1 in X_1.And denote this “$x_n \rightarrow x_1$“, as $n \rightarrow \infty$.

2.4. Definition .A C-M- Space (X_1, ρ) is complete if every “Cauchy”sequenceisaconvergent in X_1.

2.5.Definition . Suppose that two self- mappingsP_1 and $Q_1$are in aset X_1. And if “$w_1 = P_1 x_1 = Q_1 x_1$“ for some $x_1 \epsilon X_1$, then x_1 is called a “coincidence point” of P_1 and Q_1, and then w_1 is said to be a “point of coincidence” of P_1 and Q_1.

2.1. Proposition . Suppose thattwo self-mappings P_1 and $Q_1$are “O-W-C”(Occasionally –Weakly-Compatible)in a set X_1iff there is a point “x_1 in X_1” which isa“coincidence point” of P_1 and Q_1 at which P_1 and Q_1 are “commute”.

2.1. Lemma . Suppose that two self-mappingsP_1, Q_1 are (in X) “O-W-C”(Occasionally Weakly Compatible) of X_1. If P_1 and Q_1 have a unique point of” coincidence” “$w_1 = P_1 x_1 = Q_1 x_1$“, then “w_1” is a unique common “fixed point “of P_1 and Q_1.

3. MAIN RESULTS

In this main part, we prove a “unique commonfixedpoint”result for “O-W-C”(Occasionally Weakly Compatible) self - mappings in “C-Metric-Space “(Cone Metric Space). [13]

Our “main theorem” is follows:

1.3.Theorem: Suppose thatthatp$_1$ and q_1 are two self-mappings of X_1 in a “C-M- Space” (X_1,ρ) and$S_1$is a “normal cone” . And satisfying the following:

(i) $\rho(p_1x_1, q_1y_1) \leq \lambda_1 \, Max\{\rho(q_1x_1,q_1y_1), \rho(p_1x_1, q_1x_1)+\rho(p_1y_1, q_1y_1) /2\}+$

λ_2 Max$\{ \rho(q_1x_1, q_1y_1),\rho(q_1x_1, p_1x_1) +\rho(q_1y_1, p_1y_1)/2 \}$,

for all $x_1,y_1 \in X_1$, where λ_1 , $\lambda_2 >0$ and$\lambda_1 + \lambda_2 < 1$.

(ii) $p_1(X_1) \subset q_1(X_1)$

(iii) $p_1$and q_1 are O-W-C.

Then p_1 and $q_1$are having“unique common fixed point” in X_1.

Proof: By (iii) p_1 and q_1 are “O-W-C”. Then there exists a point $\alpha_1 \epsilon X_1$ such that

$p_1\alpha_1 = q_1\alpha_1 = w_1$and there exists another point $\beta_1 \epsilon X_1$ for which $p_1\beta_1 = q_1\beta_1 = u_1$.

Now we claim that : $p_1\alpha_1 = q_1\beta_1$. Suppose that $w_1 \neq u_1$. Then from (i) we get that

$\rho(p_1\alpha_1, q_1\beta_1) \leq \lambda_1 \, Max\{\rho(q_1\alpha_1, q_1\beta_1) , \rho(p_1\alpha_1, q_1\alpha_1) + \rho(p_1\beta_1, q_1\beta_1) /2\}+$

λ_2 Max$\{ \rho(_1q_1\alpha_1, q_1\beta_1),\rho(q_1\alpha_1, p_1\alpha_1) +\rho(q_1\beta_1, p_1\beta_1)/2 \}$,

$\leq \lambda_1 \, Max\{\rho(p_1\alpha_1,p_1\beta_1), \rho(p_1\alpha_1, p_1\alpha_1) + \rho(p_1\beta_1, p_1\beta_1) /2\}+$

$\lambda_2 Max\{\rho(p_1\alpha_1, p_1\beta_1),\rho(p_1\alpha_1, p_1\alpha_1) +\rho(p_1\beta_1, p_1\beta_1)/2 \}$,

$\leq \lambda_1 \, Max\{\rho(p_1\alpha_1,p_1\beta_1) ,0\}+ \lambda_2$ Max$\{ \rho(p_1\alpha_1, p_1\beta_1),0 \}$

$\leq (\lambda_1+ \lambda_2)\rho(p_1\alpha_1, p_1\beta_1)$

$<\rho(p_1\alpha_1, p_1\beta_1)$, since $\lambda_1+ \lambda_2< 1$. Which is a contradiction.

Therefore, $\rho(p_1\alpha_1,p_1\beta_1)= 0$.

Implies that, $p_1\alpha_1 = p_1\beta_1$. Therefore, $p_1\alpha_1= q_1\alpha_1 = p_1\beta_1 = q_1\beta_1 = w_1 = u_1$.

That is, $p_1\alpha_1 = q_1\alpha_1 = w_1$. Hence, "$w_1$"is a unique point of coincidence. By the (2.1)Lemma , we get that "w_1" is the "unique common fixed point " of p_1 and q_1. And this completes the proof of the theorem.

Conclusion: In this present paper, our main results aremore improve and general results than the existing results in [5].

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