**Trigonometric-Fitted One-Step 3 Points Hybrid Block Method for the Solution of Stiff and Oscillating Problems**

***Abstract***

This paper introduces a novel Trigonometric-Fitted One-Step 3 Points Hybrid Block Method tailored for addressing the complexities associated with stiff and oscillating differential equations.The continuous hybrid technique was created using the interpolation method and the collocation of the trigonometrical function as the basis function. It was then evaluated at non-interpolating points by inculcating the transformation method to produce a continuous block method. When the continuous block was assessed at each stage, the discrete block approach was regained. Upon investigation, the fundamental characteristics of the techniques were discovered to be zero-stable, consistent, and convergent. The new method is used to solve a few stiff and oscillatory ordinary differential equation problems. Comparisons of numerical results of the derived methods, it was found that our approach provides a better approximation than the existing method cited in the reference.

**AMS subject classification:** 65L05, 65L06, 65L20

**Keywords:** One-step, Hybrid Point, Transformation, Trigometrically Fitted.

**1. Introduction**

In this paper, we present a detailed exposition of the Trigonometric-Fitted One-Step 3 Points Hybrid Block Method, elucidating its formulation and elucidating the underlying principles by considering an approximate solution of second order ordinary differential equations using the one-step three offgrid point hybrid approach of the type

 

The analytic solutions are known to oscillate with known frequency which can be found in a variety of ways. Equation (1) is of particular interest to researchers due to its broad range of applications in a variety of fields, including theoretical physics and oscillatory motion, theoretical chemistry, classical mechanics,fluid dynamics, quantum mechanics, modeling scientific and engineering, celestial mechanics, and so on. Several of these problems may not be easily solved analytically and therefore there is need for construction of numerical schemes to determine the approximate results.

Several numerical algorithm integrating exactly a set of linearly independent for solution of (1) have been proposed in many work by several authors.[1] proposes a family of k-step trigonometrically fitted block falkner methods for the solution of (1), [2] proposes block trigonometrically Fitted Backward Differentiation Formula,[3] also propose a two step trigonometrically fitted method.

Among the scholars who have recently embraced the trigonometrically fitted approach in lieu of the direct method for approximating (1) are [ 4, 5, 6, 7, 8, 9, 10].

In this paper, we present a detailed exposition of the Trigonometric-Fitted One-Step 3 Points Hybrid Block Method, elucidating its formulation and elucidating the underlying principles. We showcase the method's capabilities through numerical experiments and comparisons with existing techniques, demonstrating its efficacy in solving stiff and oscillating problems. The structure of the paper is as follows: Section 2 covers the materials and techniques used in the method's development. In Section 3, the method's basis properties are analyzed, numerical experiments are conducted to test the developed method's efficiency on a few numerical examples, and the findings are discussed. Finally, we wrapped up in section 4.

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 **2. Derivation of the Method**

The continuous representation of the one step trigonometric function as the approximante solution shall be derive to generate the main method which we shall set up to obtain the block method .We consider a trigonometric approximate solution of the form

  (2)

Equation (2) is obtained by considering the trigonometric function as approximate solution and

 and are the numbers of points of collocation and interpolation, the second derivative of (2) gives

  (3)

The continuouos approximation is then constructed by imposing two conditions which are

  (4)

Collocating (3) at all points and interpolating (2) at  result to the system of non linear equation of the form

  (5)

The system of (5) is then solve to obtain the unknown parameter. By the substitutions of the values of obtained into equation (2) and using transformation from [12] gives

  (6)

 substituting (6) in (3) gives a continuous hybrid linear multistep method of the form

  (7)

We then impose (4) on in (7) and the coefficients  and 

Where 



differentiating of (7) once gives

  (8)

eveluating (7) and (8) at all points and simplifying gives the discrete hybrid block method of the form

  (9)

Where 





We obtain the following discrete scheme



**3.0** **Analysis of Basic Properties of the Method**

**3.1 Order of the Block**

According to fatunla (1991) and lambert (1973) the truncation error associated with (2) is defined by 

 (10)

Assumed that can be differentiated. Expanding (9) in Taylor’s series and comparing the coefficient of  gives the expression

 

Where the constant coefficients are given below

 

**Definition 1**: the linear operator and the associated continuous linear multistep method (10) are said to be of order  if  is called the error constant and the local truncation error is given by



For our method

Comparing the coefficient of  gives and



Hence our method is of order five (5).

**3.2 Consistency**

The One-step Hybrid trigonometrically fitted second derivative is consistent since it has order is greater than or equal to one.

**3.3 Zero Stability of Our Method**

The One-Step One Hybrid Block trigonometrically fitted fourth derivative hybrid method is said to be zero-stable if as  , the root  of the first characteristic polynomial  that is  Satisfies  and for those roots with =1, multiplicity must not exceed two.

3.4 Convergency

The compulsory terminolgy for the trigonometrically fitted to be convergent is that they must be consistent and zero-stable. Hence, our method converges since all conditions are satisfied.

3.5 Linear Stability

According to Hairer and Wanner, the concept of A-satbility is discussed by applying the test equation

 

to yield

 

where is the amplification matrix given by

 

The matrix  has eigen values  where  is called the stability function. Thus, ths stability function of our methodwith four off-grid points is given by

 

3.6 Region of Absolute Stability

The stability polynomial of our method is found to be

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**3.3 Mathematical Computation of the method**

**Problem I** We consider the stiff equation (Source: Adeniran *and Olanegan* (2019))



Exact Solution: 

Table 1 Comparison of the proposed method with Adeniran *and Olanegan* (2019)

|  |  |  |
| --- | --- | --- |
| x-values | Error in our method | Error in [3] |
| 0.1 | 8.91000E-17 | 9.99E-14 |
| 0.2 | 1.7580E-16 | 1.20 E-13 |
| 0.3 | 2.5970E-16 | 7.90E-13 |
| 0.4 | 3.4110E-16 | 1.69E-13 |
| 0.5 | 4.1920E-16 | 5.00E-13 |
| 0.6 | 4.9410E-16 | 2.00E-13 |
| 0.7 | 5.6600E-16 | 8.99 E-14 |
| 0.8 | 6.3460E-16 | 2.00 E-13 |
| 0.0 | 6.9940E-16 | 3.00E-13 |
| 1.0 | 7.6090E-16 | 2.99E-13 |

**Problem II** Consider the highly Oscilatory equation (source: Adeniran and Edaogbogun (2021))

 

Exact Solution: 

Table 2: Comparison of the proposed method with Adeniran and Edaogbogun (2021)

|  |  |  |
| --- | --- | --- |
| x-values | Error in our method | Error in [11] |
| 0.1 | 4.0000E-18 | 4.3881E-11 |
| 0.2 | 7.8000E-18 | 7.9019E-11 |
| 0.3 | 1.1500E-17 | 2.5525E-10 |
| 0.4 | 1.5200E-17 | 1.1525E-10 |
| 0.5 | 1.8800E-17 | 1.9079E-10 |
| 0.6 | 2.2400E-17 | 2.3002E-10 |
| 0.7 | 2.5900E-17 | 2.7014E-10 |
| 0.8 | 2.9300E-17 | 3.1112E-10 |
| 0.9 | 3.2700E-17 | 3.5291E-10 |
| 1.0 | 3.5800E-17 | 3.9545E-10 |

**4. Conclusion**

It is evident from the above tables that our proposed method has significant improvement over the existing methods. Trigonometric-Fitted One-Step 3 Points Hybrid Block Method is proposed for direct solution of general second order stiff and oscillatory problems where by it is self-starting when implemented. The developed method converges and is of order five.

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