**A Non-Linear Stochastic Systems on Asset Pricing Returns for Stock Market Prices.**

**Abstract**

In this paper, system of asset pricing models were used to study stock market price changes by implementing the Ito’s method; measures which could influence the volatility, the expected rate of returns and asset values which follows multiplicative effects were established through the impressions on Tables to demonstrate different price changes. In terms of generalizing the system a Geometric Brownian motion theorem were developed and proved as a deterministic concepts which does not depend on random factors. However, stochastic vector Differential Equation were considered by constraining the stochastic part of the system which is function of volatility where covariance matrix solutions were obtained to measure overall volatility or risk, by selecting assets with low or negative covariance. From the covariance matrices of each corporate investor positive and negative eigenvalues were obtained showing the assets in the portfolio are experiencing both gains and losses over time; positive eigenvalues indicates that the assets are increasing in value, which is beneficial for investors. Whereas negative values indicates that the assets are losing value, which is detrimental for investors too. More so, the mix of positive and negative eigenvalues provides important information about the risk and diversification of portfolio. To this end, a fundamental matrix were also adopted to construct portfolios that are diversified in terms of risk and predicted asset returns which is informative to investors in terms of informed decision making.

**Keywords: Stock Prices, Covariance, Stochastic Analysis , Drift, Fundamental Matrix**

**1.1 Introduction**

Stock market price changes refer to the fluctuations in the prices of stocks listed on stock exchanges. These changes can occur for a variety of reasons, including economic, political of psychological factors. Some of the key characteristics of stock market price changes include, volatility, stock prices are often volatility, meaning that they can fluctuate rapidly and unpredictably. This can make investing in stocks risky, as the value of a stock can decline sharply in a short period of time, market sentiment: the sentiment of investors and traders can have a significance impact on stock prices. In fact, financial analysis who invest in financial market are typically clueless of the behaviour of stock market; hence they go through this stock trading problem, and also faces the challenge of not knowing the type of stocks to be bought and sold for profit maximization. Hence relevant information on regular basis are required or needed by both financial analysis and potential investors for the prediction of stock price behaviour. The unstable characteristic and other significant influences like liquidity on stock returns, because the abrupt changes in share prices is erratic and happens regularly. So, in order to help investors and owners of corporations take decisions on the level of their investment in stock market [1], researchers are curious and fascinated in studying the behaviour of the unstable market variables.

Nevertheless, the price evolution of risky assets are usually modeled as a pathway or track of a risky assets that are generally of a diffusion process well-defined on several basic probability space, with the Geometric Browner motion , [2]. Numerous researchers have considered stock market prices in so many ways several ways. For instance, [3] studied systems of SDEs for economic investments whose rate of returns and asset valuation follows series price index. In the research of [4] they studied the concept of asset values with delay parameter in the model. Stochastic analysis of the behaviour of stock prices was studied by [1]. Similarly, [5] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE). [7] studied the stochastic modelling of stock prices applying a method of Brownian motion model to explain the stock prices. Hitherto [2] studied stochastic model of the fluctuation of stock market price.. However, [8] looked at a stochastic model of price changes at the floor of stock market. In the work of [9], the equilibrium price and the market growth rate of shares we determined. In another dimension [5] examined a stochastic model of several selected stocks in the Nigerian Stock Exchange (NSE) where the Euler-Maruyam method for system of (SDE) was utilized to invigorate the stock prices. In fact, lots of authors has written extensively on stock market behaviors such as, [10-14] , [15-22], [23-26] .

However, the aim of this paper is to develop stochastic system of asset pricing equations for stock market price investment plans. It is evident that investors are really affected by their personal decisions due to expected rate of returns in their investments plans. This disturbed the researchers of this paper to develop a good empirical method that can stand in terms of decisions making. It is reasonable that [26] has studied application of non-linear evolution stochastic equation with asymptotic null controllability analysis .The progress of this paper over [26] is that, this present paper models asset pricing for capital market changes where the impact analysis of volatility, return rates, asset values which follows multiplicative effects and also stating and proving of theorem which represents the behavior of asset prices and other economic variables were properly established. Our novel idea compliments previous efforts and extends frontiers in this dynamic area of mathematics of finance.

This paper is arranged as follows: Section 2.1 presents mathematical formulation, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

**2.1 Mathematical Formulation**

 In the forgoing we state the following theorem which will help solve our problems

**Theorem 2.1:(Ito’s lemma). Let ** be a twice continuous differential function on  and let  denotes an Ito’s process

  ,

Applying Taylor series expansion of  gives:

  ,

So, ignoring h.o.t and substituting for  we obtain

 

 

 

More so, given the variable  denotes stock price, then following GBM implies , the function  ,Ito’s lemma gives:

  [16] and [17].

On the other hand, the stochastic analysis on the variations stock drift and it effects in financial markets is measured. The volatility dynamics and other drift coefficients of stock prices was taken to be constant throughout the trading days. The initial stock price which is assumed to follow diverse trend series was categorized the entire origin of stock dynamics is found in a complete probability space with a finite time investment horizon. Also in real life situations there must be delay in every circumstances. Incorporating delays in cash flows or other events, investors and analysts can set more realistic expectations about the future value of an asset, which can help prevent over or under-valuations ,hence we have the following modified system of stochastic differential equations representing different rate of returns below;

  (1.1)

  (1.2)

  (1.3)

where is an expected rate of returns on stock, is the volatility of the stock , is the relative change in the price during the period of time and  is a Wiener process,  are constants and  is periodic events parameter measuring levels of return rate .

**2.2 Method of Solution**

The propose model (1.1) - (1.3) consist of a system of variable coefficient problem of stochastic differential equations whose solutions are not trivial. we solve equations independently as follows using Ito’s theorem 1.1:

From (1.1) let 

Taking the partial derivative yields

  (1.4)

According to Ito’s gives:

  (1.5)

Subtitling (1.4) into (1.5) gives

  (1.6)

 

Integrating the above expression

  (1.7)

 

Taking ln of the both sides gives

  (1.8)

From (1.2) let 

Taking the partial derivative yields

  (1.9)

According to Ito’s gives:

  (1.10)

Substituting (1.9) into (1.10) gives

  (1.11)

 

Integrating the above expression

  (1.12)

 

Taking ln of the both sides gives

  (1.13)

From (1.3) let 

Taking the partial derivative yields

  (1.14)

According to Ito’s gives:

  (1.15)

Substituting (1.14) into (1.15) gives

  (1.16)

 

Integrating the above expression

  (1.17)

 

Taking ln of the both sides gives

  (1.18)

More so, the lognormal property of stock prices can be used to provide information on the probability distribution of the continuously compounded rate of return earned on a stock between time  . If we define the continuously compound rate of return per annum realizes between time  as  ,then

  (1.19)

So that

  (1.20)

However, in order to demonstrate the empirical evidence of asset prices, we adopt Geometric Brownian Motion which is a stochastic process commonly used to model asset prices and other economic variables. The idea that it is deterministic implies that it can be expressed as a function of time , meaning that it is fully deterministic by its initial conditions and does not depend on random factors. Hence, state and prove the following theorem below.

**Theorem 2.2.** Let  be a Geometric Brownian Motion write 

show that for deterministic 

***Proof;***

Let 

where is a wiener process independent of  Then  have the same distribution, so with  So we have the RHS 

**2.3 Developing Vector valued Stochastic Differential Equation(SDE)**

Generalizing (1.1-1.3) to be SDE not modified SDE. We consider the a vector valued SDE, where the observed volatility for the real existing price processes may have some reversion drifts in the stochastic process are correlated, hence we have vector valued SDE as follows:

  (1.21)

where  represents asset price, is drift which is the return rates of underlying assets, is volatility processes, see references therein [4],[24-25]. Since both processes S1,…,Sn are correlated, to develop a vector equation for ( 1.21 ) write the equations in matrix form as follows:

  (1.22)

A generalized equation for the vector valued SDE can now be put in the form

  (1.23)

Where $A\left(t\right)\in R^{n×n},B\_{i}\left(t\right)\in R^{n×n},w\_{i}\left(t\right)\in R^{n}$ is an n-dimensional Brownian motion, $x$(t)$\in R^{n}$

It is known in [24], that $x$(t) for equation (1.23) is normally distributed because the Brownian motion is just multiplied by time-dependent factors.We compute the stochastic integral solution for the equation (1.23) as follows. Let $X(t)\in R^{n×n}$ be fundamental matrix of the homogenous stochastic differential equation (1.23). It is assumed that x(t) is a continuously differentiable function in t, with

 X(t) = 0, t < 0

 I, t = 0 (I = identify)

Then the solution of (1.23) is given by



Setting the Right Hand Side of(1.23) to zero yields as follows

  (1.24)

However, analyzing the determinants, investors can gain valuable insight into asset returns and make more informed investment decision. Therefore, we have the following asset returns from the three asset pricing models as follows:

  (1.25)

  (1.26)

  (1.27)

Nevertheless, by combining asset returns of different assets in other to reduce risk; spreading investment across various assets. Hence, we integrate the functions of each asset returns above as:

  (1.28)

  (1.29)

  (1.30)

**3.1 Results and Discussion**

This Section presents the table results for whose solutions are in (1.8), (1.13) and (1.18) . Parameter values were choosing on the basis of reflecting the actual characteristics of the asset to ensure the valuation model is realistic and captures the true value of the asset. Hence the following parameter values were used in the simulation study:



 **Table 1: The result of volatility on the measure of asset values when time is constant**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|   |   | Volatility  |   |   |   |   |  |
| 60.77 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 60.1753.3645.4837.23 | 64.5362.6251.7844.02 | 674.51258.2682.7721.22 | 1.691,641.571.49 | 0.990.650.25-0.23 |
| 50.25 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 50.7645.0238.3630.78 | 53.3648.6442.8236.40 | 470.54180.0856.3914.16 | 1.951.891.811.70 | 1.110.690.19-0.38 |
| 40.10 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 39.7035.2130.0024.57 | 42.5838.8234.1729.05 | 293.78112.3235.419.09 | 2.302.222.121.99 | 1.250.720.10-0.62 |



**Table 2: The result of volatility on the measure of asset values when time is constant.**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|   |   | Volatility  |   |   |   |   |  |
| 60.77 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 60.1753.3645.4837.23 | 64.5362.6251.7844.02 | 28,700.1410,987.3123,498.0315,748.66 | 2.462.412.343.42 | 1.771.421.801.72 |
| 50.25 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 50.7645.0238.3630.78 | 53.3648.6442.8236.40 | 19,623.957,511.0616065.6710,769.84 | 2.882.822.742.64 | 2.041.622.081.98 |
| 40.10 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 39.7035.2130.0024.57 | 42.5838.8234.1729.05 | 12,495.964,783.6110,783.616,857.42 | 3.463.393.293.16 | 2.421.512.472.34 |



**Table 3: The result of volatility on the measure of asset values when time is constant**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|   |   | Volatility  |   |   |   |   |  |
| 60.77 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 60.1753.3645.4837.23 | 64.5362.6251.7844.02 | 28,131.2910,833.2823,031.5515,438.49 | 2.452.402.349.02 | 1.761.421.791.71 |
| 50.25 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 50.7645.0238.3630.78 | 53.3648.6442.8236.40 | 19,233.247,362.3315,747.3610,555.22 | 2.872.812.732.63 | 2.041.622.081.98 |
| 40.10 | 4.00004.00004.00004.0000 | 0.50.60.70.8 | 39.7035.2130.0024.57 | 42.5838.8234.1729.05 | 12,248.884,690.6410,028.56,722.74 | 3.463.383.283.16 | 2.411.892.462.34 |

In Tables 1, 2 and 3: shows increase in volatility decreases the value of asset; this means that the asset is more likely to experience large price swings in either direction. When this happens, the asset’s value can be said to have decreased because its price is more uncertain. This makes the asset less desirable for investors, who typically prefer assets that have stable, predictable prices. As a result, increased volatility can lead to a decline in the value of an asset.

Then the multiplicative effects of two asset values refers to the impact that changes in the values of two assets can have on the overall value of a portfolio or investment. If a portfolio is heavily weighted towards a few assets that have highly correlated prices, the multiplicative effect of changes in these asset prices can significantly impact the value of the portfolio, see column 6 of Tables 1,2 and 3. However, in terms of comparing the models which produced small or big asset values as the best cannot really be determined, for instance, if an investor is risk-averse, it may prefer a model that produces smaller asset values that are less volatile and less susceptible to large swings in price. On the hand, if an investor have a long-term investment horizon, it may be more willing to accept higher volatility in the short term in exchange for the potential for higher returns over time. More so, big and small returns seen in columns 7 and 8 of the three tables above refers to the magnitude of changes in the value of an investment over time. These large positive or negative changes in the value of an investments, for instance, a stock that increases in value by 50% in a single year would be considered to have a big return. The smaller positive or negative changes in the value of an investment, for instance, a stock that increases in value by 5% in a single year would be considered to have a small return.

,

The eigenvalues of the above matrices for first Asset values  expected return rates as follows:





The eigenvalues of the above matrices for second Asset values  expected return rates as follows:



The eigenvalues of the above matrices for third Asset values  expected return rates as follows:



Here, asset that have low covariance or even negative covariance can provide diversification benefits for investors, as they tend to perform differently in different market conditions. This means that a portfolio that includes a mix of assets with low covariance can be less risky than a portfolio that includes only assets with high covariances. The covariance matrix are used to construct portfolios that have low overall volatility or risk , by seleting assets with low or negative covariance. On the aspect of positive and negative eigenvalue of returns implies having such results indicates that the assets in the portfolio are experiencing both gains and loses over time. Positive values indicates that the assets are increasing in value, which is beneficial for investors. while negative values indicates that the assets are losing value, which is detrimental for investors.The mix of positive and negative values provides important information about the risk and diversification of portfolio.

The fundamental matrix solutions of equation (1.24) can be found following the methods of [23-24] and [26]







Finding the determinants and integrations of asset returns respectively

 is by applying (1.25-1.30) as follows:







The predicted asset returns above where there is positive and negative values: Positive returns provides investors with capital gains, while negative returns represent losses. This reflects the inherent risk and reward tradeoff in investing. Now, that the B’S and C’s asset returns contains a mix  of assets with positive and negative returns, it will diversified and more resilient to market volatility than a portfolio that contains positive or negative returns.

The determinant of asset returns above provides a measure of changes in value of a portfolio over a given period of time. It’s also a way of quantifying the effects of market movements of each independent asset returns on portfolio performance. On the other hand, the integral results of asset returns provides a measure of the total return of portfolio over a given period of time; which involves calculating the sum of the returns of assets in the portfolio. This scenario can help investors assess the performance of portfolio of investments.

**Table 4: Predicted Asset Returns of the three independent Investors**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | Mean | Kurtosis | Skewness |
|   | 43.68 | 19.97 | 41.11 | 34.92 | 1.5000 | -0.6762 |
|  | -510.81 | -231.89 | 310.51 | -144.06 | 1.5000 | 0.3692 |
|  | -502.72 | -92.79 | 225.94 | -123.19 | 1.5000 | -0.1518 |

In the above Table, the mean values provides a more conservative estimate of stocks which can help investors manage risk and make more informed investment decisions, see column 5. The kurtosis is a measure of the ‘’flatness’’ or peakedness of a distribution, where there are more extreme values on one side of the distribution than the other. If the distribution is right skewed, it means that there are more high values in the predicted prices, see column 6. In the vain, the skewness is related measure of distribution, indicating the degree to which a distribution is asymmetric. A positive skewness with more high values than the low values. This indicates that investors are bullish and expected prices to rise over time see column 7.

**4.1 Conclusion**

This paper studied, system of asset pricing models were used to study stock market price changes by means of Ito’s method; measures which could influence the volatility, the expected rate of returns and asset values which follows multiplicative effects were established through the impressions on Tables to demonstrate different price changes. In terms of generalizing the system a Geometric Brownian motion theorem were developed and proved as a deterministic concepts which does not depend on random factors. However, stochastic vector Differential Equation were considered by constraining the stochastic part of the system which is function of volatility where covariance matrix solutions were obtained to measure overall volatility or risk, by selecting assets with low or negative covariance. From the covariance matrices of each corporate investor positive and negative eigenvalues were obtained showing the assets in the portfolio are experiencing both gains and losses over time; positive eigenvalues indicates that the assets are increasing in value, which is beneficial for investors. While negative values indicate that the assets are losing value, which is detrimental for investors too. More so, the mix of positive and negative eigenvalues provides important information about the risk and diversification of portfolio. To this end, a fundamental matrix were also adopted to construct portfolios that are diversified in terms of risk and predicted asset returns with some statistical variations like mean, kurtosis and skewness gave significant meaning which is informative to investors in terms of informed decision making. Consequently, we recommend stability analysis on stochastic differential equations with control studies in the next study.

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