|  |  |
| --- | --- |
|  | |
| Journal Name: | [Asian Journal of Pure and Applied Mathematics](https://www.jofmath.com/index.php/AJPAM) |
| Manuscript Number: | **Ms\_AJPAM\_1981** |
| Title of the Manuscript: | **A Transformative Geometric Framework for Dihedral Groups: The Symmetry Density Index in Three-Dimensional Space** |
| Type of the Article |  |

|  |  |  |
| --- | --- | --- |
| **PART 1: Comments** | | |
|  | **Reviewer’s comment**  **Artificial Intelligence (AI) generated or assisted review comments are strictly prohibited during peer review.** | **Author’s Feedback** (It is mandatory that authors should write his/her feedback here) |
| **Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.** | This manuscript introduces the Symmetry Density Index (SDI), a bold metric for analyzing dihedral group actions in three-dimensional (3D) space. The work attempts to bridge a gap in symmetry group theory from classic planar applications to the volumetric/spatial settings. This could have significant implications for many fields, from computational topology to quantum chemistry, robotics, and materials science. The SDI concept is appealing and, if its utility is demonstrated in more practical contexts, could be a valuable tool for interdisciplinary research. | The scientific community gets high benefits from the application of the dihedral group.The research paper deals with symmetry density index ( SDI) under the various dimensions. It deals with the application of rotations and reflection. |
| **Is the title of the article suitable?**  **(If not please suggest an alternative title)** | The title is accurate and descriptive. For clarity, may be considered some note on the focus on 3D volumetric analysis, for example: "A Transformative Geometric Framework for Dihedral Groups: The Symmetry Density Index for Three-Dimensional Volumetric Analysis" | The title in itself define the research paper. The title is complete.  “A transformative Geometric Framework for Dihedral Group: The Symmetry Density Index in Three- Dimensional Space. |
| **Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.** | The abstract presents the goals and contributions of the manuscript. However, it would benefit from mentioning limitations or open questions, such as sensitivity of SDI to the choice of symmetry axes or the challenges in applying the method to irregular, non-convex, or real-life objects. Including a brief concrete example from the computational experiments could make the abstract more tangible. | Abstract of the research paper is the summary of the research paper. Abstract is written in the same way in which research paper is written. I don't think there is a need for modification in the abstract. |
| **Is the manuscript scientifically, correct? Please write here.** | The manuscript appears mathematically rigorous, providing clear definitions, few theorems with proofs, and illustrative examples for canonical shapes (cube, sphere, torus, etc.). However, several important methodological and practical aspects need clarification or further development:   1. There is no systematic or algorithmic method described for the selection of symmetry axes, particularly for objects possessing multiple, ambiguous, or continuous axes (e.g., polygons, polyhedral, or objects with helical/spiral structure). This could significantly affect the outcome, and is especially problematic for non-trivial geometries. 2. I have doubts about the practical applicability of the method in highly irregular, non-convex, or biologically relevant shapes (protein structures or DNA helices), where the definition of symmetry may be local or context-dependent. The manuscript would benefit from practical guidance or at least a discussion of limitations. 3. Applications to quantum chemistry and quantum topology are mentioned, but the methodology for applying SDI to quantum superposition, electron density distributions, or non-classical symmetries remains unclear. For instance, does SDI have any potential to distinguish between different topological or entangled states in quantum systems, or does the classical nature of the metric limit its usefulness? This is unclear and deserves a brief discussion. 4. There is no discussion of computational complexity or feasibility of applying method to large or data-rich systems (e.g., in real-world computational chemistry or network science). 5. No algorithmic pseudocode or reproducibility guidance is provided.   In summary, the theory is robust **for idealized** objects, but the manuscript lacks depth in practical implementation, real-world applications, and computational detail, especially for complex, irregular, or quantum-mechanical systems. | SDI enhances 3D object recognition by identifying symmetry-dense regions, improving robustness in occlusion-heavy scenes. In dynamic settings, SDI tracks symmetry evolution over time, aiding motion estimation in autonomous navigation systems.  The computational results reveal distinct SDI behaviors: compact objects like the tetra-hedron yield lower values due to tight orbits, while larger, symmetric structures like the dodecahedron and torus exhibit higher SDI, reflecting denser symmetry distributions.  The sphere’s constant SDI suggests a limitation in centered configurations, prompting exploration of off-center p0 in future studies. |
| **Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.** | The references are appropriate and cover the main foundational literature in group theory and geometry (Artin, Coxeter, Armstrong, etc.). However, for a work with claimed interdisciplinary applications, it would be beneficial to cite more recent literature from areas such as:   1. Topological data analysis 2. Symmetry detection in molecular/biological structures 3. Computational approaches in group theory and/or graph theory 4. Practical applications in computer vision or data science | In my opinion the references are sufficient and lucid. I am going to include latest research in the new manuscript. |
| **Is the language/English quality of the article suitable for scholarly communications?** | The manuscript is written in clear, formal academic English and is accessible to mathematically literate readers. | English language easily accessible. |
| **Optional/General** comments | I strongly encourage the authors to provide more explicit algorithmic guidance or at least heuristic recommendations for the selection of symmetry axes and reference points in practical cases.  The manuscript would be greatly improved by including at least one realistic example involving a highly irregular, biological, or noisy dataset (e.g., a protein, a helical nanostructure, or simulated quantum system), with a step-by-step SDI calculation and interpretation.  A discussion of the computational complexity, robustness to noise, and practical interpretability of SDI values would add significant value.  Providing open-source code/ pseudocode would be a significant benefit for reproducibility and interdisciplinary adoption.  More discussion on the potential and limitations of SDI in quantum topology and quantum chemistry would clarify its scope. For instance, does the SDI capture any meaningful features of quantum superpositions or only classical spatial symmetries? What are the prospects for extending SDI to account for quantum uncertainty or probabilistic densities? | The various pictures easily illustrate the research paper theoretically as well as diagrammatically. It is written in the way to express the application of the dihedral group. The various reflections and rotation clearly define the research at the topmost level. |

|  |  |  |
| --- | --- | --- |
| **PART 2:** | | |
|  | **Reviewer’s comment** | **Author’s Feedback** (It is mandatory that authors should write his/her feedback here) |
| **Are there ethical issues in this manuscript?** | *(If yes, Kindly please write down the ethical issues here in details)* | No ethical issues |