**Solution of Certain System of Ordinary Differential Equations using “Saxena & Gupta Transform”**

**Abstract:** Integral transforms play an important role in solving system of ordinary differential equations and integral equations. Integral transformation are essential for solving complex problem in business, engineering, natural sciences, computers, optical science, and modern mathematics. In the present paper we discuss some applications of new transform “Saxena & Gupta” transform is an interesting method to solve certain type of system of ODEs. The main advantage of using “Saxena & Gupta” transforms to solve ordinary differential equations is that they convert differential equations into algebraic equations, which are often easier to solve. This method is particulary useful for linear ODEs with constant coefficients and initial conditions, as it simplifies the process and avoids the need for complex integration techniques.

**Keywords** : Saxena & Gupta transform, inverse Saxena & Gupta transform, system of differential equation, Boundary value problems.

1. **Introduction**

Fractional calculus is the branch of mathematics which deal with the investigation and applications of integrals and derivatives arbitrary order. Due to the growing range of applications, there has been significant interest in developing transforms for the solution of fractional differential equations.

Integral transforms are the most useful techniques of the mathematics which are used to find the solutions of differential equations, partial differential equations, integro-differential equations, partial integro- differential equations, delay differential equations and population growth.

In this paper we apply a new integral transform, called Saxena & Gupta transform, for solving a system of ordinary differential equations. Integral transformations essential for solving complex problems in engineering, natural sciences, computers, optical sciences, and modern mathematics to a simple system of algebraic equations that can be solved easily. We can use some common methods for solving higher-order boundary value problems of ordinary differential equation. We can convert ODE into an algebraic system of equations by Hermite Wavelet Method and find common approximate solution [12],[13].

**Definition:**

**Let**  be a function of . The new transform of a function is defined as follows, see in [ 7 ],

... (1.1)

The above integral is convergent.

**Saxena & Gupta transform of derivatives:**

**Table 1: Saxena & Gupta transform of some elementary function.**

|  |  |  |
| --- | --- | --- |
| S.NO. | Function | New transform |
| 1 | 1 |  |
| 2 |  | 1 |
| 3 |  | 2ʋ |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |

1. **Application of Saxena and Gupta Transform of Certain system of Ordinary Differential Equations.**

As specified in the introduction of this paper, the Saxena and Gupta transform can be used as an effective tool. For analysing the basic properties of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of the Saxena and Gupta transform in solving certain initial value problems described by system of ordinary differential equations [1], [2], [3].

**Theorem (2.1):** Consider the system of differential equations,

.

... (2.1.1)

… (2.1.2)

with initial condition

**Solution:** To obtain the solution of system of ordinary differential equations first weApplying the Saxena & Gupta transform of both side of eq. (2.1.1) and (2.1.2)

since

+ ...(2.1.3)

- …(2.1.4)

Solving this equations for

… (2.1.5)

… (2.1.6)

Applying inverse Saxena & Gupta transforms,

since

Thus required solution of given differential equations are

... (2.1.7)

… (2.1.8)

**Theorem (2.2):** Find the solution of the system of the differential equations,

… (2.2.1)

.... (2.2.2)

with initial conditions , where are arbitrary constants.

**Solution:**To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.2.1) and (2.2.2), we get

since

+ …(2.2.3)

.

Solving this equations for

…. (2.2.5)

… (2.2.6)

Applying the inverse Saxena & Gupta transform both sides of the equation (2.2.5) and (2.2.6).

since

thus required solution of given differential equations are

…. (2.2.7)

... (2.2.8)

**Theorem (2.3):** Find the solution of the system of ordinary differential equations

… (2.3.1)

...(2.3.2)

with initial conditions

**Solution**: To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.3.1) and (2.3.2), we get

since

... (2.3.3)

+2 …(2.3.4)

Solving this equations for

... (2.3.5)

... (2.3.6)

Applying the inverse Saxena & Gupta transform both sides of the equation (2.3.5) and (2.3.6)

since

thus required solution of given differential equations are

… (2.3.7)

... (2.3.8)

**Theorem (2.4):** Find the solution of the system of ordinary differential equations

… (2.4.1)

.... (2.4.2)

with initial conditions

**Solution:**To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.4.1) and (2.4.2).

since

... (2.4.3)

+ ... (2.4.4)

Solving this equations for

... (2.4.5)

... (2.4.6)

Applying the inverse Saxena & Gupta transform both side of the equation (2.4.5) and (2.4.6)

since

thus required solution of given differential equations are

… (2.4.7)

…. (2.4.8)

**Theorem (2.5):** Find the solution of the system of ordinal differential equations

… (2.5.1)

.... (2.5.2)

with initial conditions

**Solution:** To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both side of eq. (2.5.1) and (2.5.2)

since

... (2.5.3)

=2 ... (2.5.4)

Solving this equations for then applying inverse transforms we get the solution of given differential equations are

… (2.5.5)

…. (2.5.6)

**Theorem (2.6):** Find the solution of the system of the equations

… (2.6.1)

.... (2.6.2)

With initial conditions

**Solution:** To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.6.1) and (2.6.2)

)

since

... (2.6.3)

… (2.6.4)

Solving this equations for

…. (2.6.5)

... (2.6.6)

Applying the inverse Saxena & Gupta transform both side of the equation (2.6.5) and (2.6.6)

since

thus required solution of given differential equations are

…. (2.6.7)

…. (2.6.8)

**Importance:**  Saxena & Gupta Transform provide a powerful and systematic method for solving ordinary differential equations by transforming them into simpler algebraic equations, making them a valuable tool in various scientific and engineering applications. This method is widely used in various fields like engineering, physics, and applied mathematics.

**Conclusion:** This innovative technique demonstrates greater effectiveness and ease of use in handling ordinary differential equations compared to conventional methods. Also this method is very efficient, simple and engineering applications , with the potential to extend its utility to a wide array of problems across various domains. The main goal of this research is to solve certain system of ordinary differential equations.

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1.

2.

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