**EUROPEAN PUT OPTION VALUATION WITH STATISTICAL TESTS FOR CAPITAL MARKET INVESTMENTS.**

**ABSTRACT**

The analysis of European put option implies contracts which permit investors to sell particular number of securities within specified time frame at a predetermined price. In particular, the Black-Scholes put option were investigated on the share prices of Fidelity, Access and Merged Banks which paved way to obtain put option close form prices. The table results were presented on disparities of put option prices at specified time frame and the effect of the relevant parameters were discussed. Also, the share prices were subjected to statistical test using Bartlett’s test, Fisher’s F-max test and Cocharan’s C-test. The results show that the variances of the shares prices are different from Bartlett and Fisher tests while Cocharan’s C-test was on the contrary which informs investors to make some vital decisions. The results presented here will also be beneficial to banks managements for decision making depending on their share price changes.

**Keywords: European put option, share price, investment, capital market and Banks**

* 1. **INTRODUCTION**

The stock market been highly volatile isn’t a question for debate. Many investors have long sought for mathematical models that could almost if not correctly predict future price fluctuation. Economists such as [1] developed a mathematical model for financial markets which takes cognizance of derivative instruments. This model became popularly known as the Black-Scholes model. This have paved way for other Researchers to come up with meaningful mathematical and statistical models following their assumptions in handling the fluctuation that exist in the stock market. The first mention of the ''Black-Scholes” is attributed to Robert Merton with options pricing model in 1973, which enables option marketers to price options on all types of financial market almost correctly. This noble achievement won Robert C. Merton and Myron Scholes the Nobel Memorial Prize in Economic Studies 1997.European-style options-which case can only be executed on the cessation date. Whereas in the American-style options it is possible to exercise the option at any time during the life of the option.

Traders that believe the underlying stock will go up over time buy these call options hoping for profits. On the flip-side, put options give the option holder the right to sell the underlying stock for an agreed upon price anytime between today and upon expiration. Traders that think a stock is going to go down can buy these put options in the hopes of making money if the stock goes down.

Conversely, numerous researchers have applied Black-Scholes model in different approaches; for stance, [2] considered the impact of Crank-Nicolson Finite Difference approach in evaluating Options. [3] was of the view that the rate of the option lies on the underlying asset, which is frequently a stock, commodity, currency or an index. In a similar way [4] established a new technique of assessing pricing effects on the premise to reduce pricing bias. [5] used the Tempered fractional derivative to price a European-double-knock-out barrier option. This study analyzed characteristics of three fractional Black-Scholes models through comparison with the classical Black-Scholes model. [6] examined Black-Scholes model analysis and violated the assumptions of BS that says volatility is constant.

However, [7] showed that the Black-Scholes model has been a major advance in finance over a period of time. [8] posited that since the Black–Scholes Option pricing model (BSOPM) has long been in use for valuation of equity options to find the price of stocks. [9] proposed a high accurate method based on non-standard Runge–Kutta (NRK), modified weighted essentially non-oscillatory. [10], the Laguerre neural network was proposed as a novel numerical algorithm with three layers of neurons for solving BS equations. [11] proposed a framework based on the celebrated transform of Mellin type (MT) for the analytic solution of the Black-Scholes-Merton European Power Put.[12] examined the problem of pricing the double barrier option in a Black-Scholes environment, in which the volatility undergoes random jumps at random times, and obtained a closed-form solution for the option price as a power series. Lots of scholars have extensively done work on Black-Scholes such as [13-23] etc.

The analytical approach of solving BS equation for option pricing is not a difficulty one.

The great encounter in this paper is the ability of coming up with some realistic assumptions that will be analyzed to fit financial market. So, we propose an analytical approach to Black-Scholes to give close form put option prices of Fidelity, Access and Merged banks. This merged bank was discovered by [14] in studying the movement of share prices through transition probability matrix but our case is to value this share prices via put option. The share prices were subjected to statistical test using Bartlett’s test, Fisher’s F-max test and Cocharan’s C-test. The results show that the variances of the shares prices are different from Bartlett and Fisher tests while Cocharan’s C-test was on the contrary which inform investors or bank management. This noble idea has not been seen elsewhere. Therefore, this extends the works of [13] and [14] respectively. The aim of this paper is to examine European put option prices with some statistical test’s variations for capital market investments.

The plan of this paper is set as follows: Section 2.1is Mathematical preliminaries, Section 3.1 Results and Discussion and conclusion are seen in Section 4.1.

**2.1 Mathematical preliminaries**

Here we state some few definitions which forms the basis of this paper.

**Definition 1: Stochastic Processes :** Stochastic process: A stochastic process is a relation of random variables , i.e. for each  in the index set is a random variable. Now we understand  as time and call the state of the procedure at time t. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

It can also be seen as a statistical event that involves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defined at a set of time points which may be continuous or discrete.

**Definition 2: Random Walk**: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point to a point . A random walk is a stochastic sequence with , defined by

(1)

Where are independent and identically distributed random variables.

**Definition 3: Brownian Motion: **The Brownian motion or Wiener process is a basic process that serves as part of many different processes.

It refers either to: (i) The physical event that small particles immersed in a fluid go randomly;

(2) The Mathematical tool used to explain those random activities. It is the scaling bound of random walk in one aspect as the time steps to zero that is the quantity of steps becomes large.

Mathematically, Brownian motion is a set of random variables, each value of the real t in interval,

This set has the subsequent characteristics:

is continuous function of the parameter  ,with

For each ,  is normally distributed with expected value 0 and t independent of each other.

Conversely, in order to use a Brownian Motion as a Model for stock prices, at time **** we prefer the stock returns over the period to have a constant drift, and volatility, .

**Definition 4: Stochastic Differential Equation (S.D.E):** A stochastic differential equation is a differential equation with stochastic term. let  be a probability space with filtration  and  an m-dimensional Brownian motion on the given probability space. We have a stochastic differential equation in coefficient functions of f and g as follows:



Where is an n-dimensional random variable and coefficient functions are in the form and . SDE can also be written in the form of integral as follows:

# 



Where  are terms known as stochastic differentials. The is a valued stochastic process.

However, let  be the price of some risky asset at time , and  ,an expected rate of returns on the stock and  as a relative change during the trading days such that the stock follows a random walk which is govern by a stochastic differential equation.

 (2)

**Theorem 1.**: (Ito’s formula) Let  be a filtered probability space be an adaptive stochastic process on  possessing a quadratic variation with SDE defined as:



 and for



Using theorem 1 and equation (2) comfortably solves the SDE with a solution given below:



**2.2 Mathematical Framework of Black-Scholes Model**

The seminar paper on option pricing by Fisher Black and Myron Scholes in 1973 (Black and Scholes 1973) has had great impact on contemporary research in Mathematics of finance. This model is commonly used in financial modeling. The Black-Scholes model is made up of seven assumptions:

* The asset price has characteristics of a Brownian motion with and  as constants.
* The transaction costs or taxes are not allowed.
* The entire securities are absolutely divisible.
* Dividend is not permitted during the period of the derivatives.
* Unacceptable of riskless arbitrage opportunities.
* The security trading is constant.
* The option is exercised at the time of expiry for both call and put options.

In mathematical finance, an arbitrage arguments show that any derivative  written on  must satisfy the partial differential equation of the form of option pricing; hence we have the following:

:

 . (3)

 (4)

 . (5)

And final time condition given by :

 (6)

Where  represents interest rate,  represents volatility of the underlying assets and  represents time of maturity.

With boundary conditions:

Equation (5) is the value of asset which is worthless when the stock price is zero. The details of the above option model can be expressly found in [1],[2] and [13] etc.

To eliminate the price process in (3) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

 (7)

where is Price of a call option, is price of underlying asset,  is the strike price,  is the riskless rate, is time to maturity, is variance of underlying asset, is standard deviation of the underlying asset (generally referred to as volatility), and is the cumulative normal distribution. Similarly Black-Scholes analytic formula for the prices of European Put option is given as follows

 (8)

Where P is the price of put option and the meaning of other parameters remain the same as in [1-2].

**2.3 Statistical Tests in Analyzing the Variances of Shares Prices: Bartlet’s test, Fisher’s max test and Cochran’s test.**

**2.3.1. Bartlett’s Test.**

Hypothesis

(All the variances are homogeneous)

(At least one of the variance is different)

Test statistic: )(2

Where and

For equal n

Where for unequal n

For equal n, the statistic can be simplified as follows

is the sample variance of the ith share prices of the banks is defined below;

Decision criterion

Reject the null hypothesis if )(2 > )(2 the upper percentage point of the chi-square distribution with degrees of freedom and level of significance

**2.3.2 Fisher’s F-max Test**

(All P variances are homogenous)

(At least one of the p variances is different)

Test Statistic

Where S2max is the largest variance and S2min smallest variance

Decision criterion.

Reject the null hypothesis if Fmax > Fmax  Fmax  is the upper and percentage point of Fmax table critical values

**2.3.3 Cochran’s C- Test**

Hypothesis

(All the P share price variances of Fidelity, Access and Merged banks, are all equal)

(At lest one of the share price variances of fidelity, access and Merged bank is different)

Test statistics

S2Max is the largest variance and

Decision Criterion

Reject the null hypothesis if C > C () where C () is the upper percentage point of C table of critical values.

**3.1 RESULTS AND DISCUSSION**

In this Section we present the simulation results for the problem in Section 2.1. The table results are implemented in MATLAB programming language.

**Table 1: The value of share price of Access Bank, Plc and disparities of maturity days:**

|  |  |  |
| --- | --- | --- |
| Initial share price | Put option prices when time t=4 | Put option prices when time t=6 |
| 410 | 398.7668 | 402.4696 |
| 80 | 68.4077 | 71.1419 |
| 126 | 114.7326 | 118.1743 |
| 79 | 67.3878 | 70.0954 |
| 98 | 86.6285 | 89.7471 |
| 92 | 80.5775 | 83.5869 |
| 127 | 115.7342 | 119.1833 |
| 91 | 79.5673 | 82.5567 |
| 378 | 366.7668 | 370.4692 |

As can be seen in Table 1, when the maturity time of a put option on the share price of Access Bank increase, the value of the put option also increase. Theoretically, this is because time value of the put option, which is the difference between the current value of the option and its intrinsic value, increases as maturity time increase, the put option holder has more time to potentially benefit from a decrease in the share price.

The volume of the initial share price of access bank can indeed affect the value of a put option on the share. In order to determine the value of a put option, one must first calculate the fair value of the share. This value is determined by taking into account the current share price, as well as the expected future value of the share.

**Table 2: The value of share price of Fidelity bank, Plc and disparities of maturity days.**

|  |  |  |
| --- | --- | --- |
| Initial share price | Put option prices when time t=4 | Put option prices when time t=6 |
| 415 | 403.7668 | 407.4697 |
| 62 | 49.8105 | 51.9957 |
| 138 | 126.7473 | 130.2642 |
| 61 | 48.7572 | 50.9091 |
| 121 | 109.7232 | 113.1249 |
| 81 | 69.4266 | 72.1867 |
| 139 | 127.7482 | 131.2702 |
| 80 | 68.4077 | 71.1419 |
| 384 | 372.7668 | 376.4693 |

It can be seen be in Table 2 that has helped put option values than Fidelity bank It could suggest that investors perceive access bank to be riskier than fidelity bank, and are willing to pay more for the right to sell its shares at a set price in the future. Alternatively. It could mean that the current share price of access bank is higher than fidelity bank, or that of access bank's volatility is higher.

In either case, this could have implications for the company's future performance and the value of its shares. It could be with dipping deeper with into the data to understand; see Table 2

**Table 3: The value of share prices when the two Banks merge with disparities of maturity days**

|  |  |  |
| --- | --- | --- |
| Initial share price | Put option prices when time t=4 | Put option prices when time t=6 |
| 825 | 813.7668 | 817.4701 |
| 142 | 130.7506 | 134.2872 |
| 264 | 252.7662 | 256.4616 |
| 140 | 128.7490 | 132.2761 |
| 219 | 207.7661 | 211.4464 |
| 173 | 161.7626 | 165.3936 |
| 266 | 254.7666 | 258.4620 |
| 171 | 159.7622 | 163.3894 |
| 762 | 750.7668 | 754.4701 |

In Table 3, the two banks are merged which result to several financed implications. For instance, the merged entity would likely have a larger market capitalization, or value, then either of the individual banks. This could make it easier for the merged bank to raise capital such as through a stock offering. Additionally, the merged bank might be able to take advantage of economies of scale and synergies, such as sharing back office functions, which could reduce costs and improve efficiency. Finally, the merger could lead to increased computation in the banking sector which could benefit consumers.

**Table 4: The share price data of Fidelity, Access and Merged Banks**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Number of observation | | | | | | | | |
| Banks | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Fidelity | 415 | 62 | 138 | 61 | 121 | 81 | 139 | 80 | 384 |
| Access | 410 | 80 | 126 | 79 | 98 | 92 | 127 | 91 | 378 |
| Merged | 825 | 142 | 264 | 140 | 219 | 173 | 266 | 171 | 762 |

**Applying subsection 2.3.1 of Bartlett’s Test for The Three Banks.**

(All the three shares price variances Fidelity, Access and Merged Banks are homogeneous)

(At least one of the three share price variances are different)

for unequal n.

s

And

Thus, the Barttlet’s test statistic is

)(2

Decision.

Since )(2 = 16.6 > )(20 .05 = 5.99 we reject the null hypothesis.

Conclusion: The variances of the share price for the three banks are different.

**Applying subsection 2.3.2 of Fisher’s F-max Test for The Three Banks.**

Following the hypothesis under fisher’s F-max test above:

(The variances of the share prices for Fidelity, Access and merged banks are homogenous)

(At least one of the variances of the share price for Fidelity, Access and merged banks is different)

Test statistic

Fmax (0.05, 3, 26) = 2.98

Since Fmax = 5.00 > 2.98 we reject the null hypothesis.

Conclusion: The variances of the three banks share price are different.

**Applying subsection 2.3.3 of Cochran’s C- Test for The Three Banks**

(The variances of the share prices for Fidelity, Access and Merged banks are equal)

(At lest one of the variances of share price for fidelity, Access and Merged bank is different)

Test statistics

Since , the null hypothesis cannot be rejected.

Conclusion: the variances of the share prices for the three banks are homogenous.

This conclusion vary with the one gotten from Bartlet’s and fisher’s F-max test.

In all Bartlett’s and fisher’s test are both used to test for equal variances in the share prices of fidelity, access and merged banks. These tests are showing that the variances are different. Cochran’s C-test, on the other hand, is showing that the share price is homogenous despite the different variance.

These results indicate that while the share prices have different levels of volatility, they are still closely related in some ways, such as being influenced by the same economic or managerial factors of the banks. However, the fact that Bartlett’s test and fisher s’ text are both indicating that the variances are different, C- test is showing homogeneity which could be influenced by the power of the test. Bartlett’s and fisher’s test are providing a more accurate representation of the share price data.

**4.1 Conclusion**

This paper considered analytical solution of Black-Scholes on share price of Fidelity, Access and Merged Banks. From the study the disparities on the specified time frame is obtained effectively which shows that increase in specific time frame increases the value of put option for the three banks under study. Also, the share prices were subjected to statistical tests using; Bartlett’s test, Fisher’s F-max test and Cocharan’s C-test. The results show that the variances of the share prices are different from Bartlett and fisher’s tests while Cocharan test shows that the variances are homogenous despite the different variances which is informative to the bank management in running day-to-activities. In the next study we shall be looking at Black-Model with non- parametric properties,

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