Stochastic Models for Capital Asset Pricing: Impact of Delay Parameters on Asset Valuation

**Abstract**

In finance generally, investments are ventures associated with risk which cannot be avoided in daily lives. Therefore this paper considered stochastic systems with importance on disparities of stock quantities by implementing the Ito’s method of solution where certain measures were given on the assessments of asset values with and without delay parameters. Consequently, the impressions on Tables for investors in financial markets were analyzed to establish empirically the characteristics of asset values with and without delay parameters when volatility increases. From the stochastic analysis of the problem we presume that ; increase in volatility decreases the value of assets, incorporating delay parameter increases the value of assets, the asset values with delay is realistic than those that do not for a few reasons; More so, the probability normal plot shows positive linear trend which suggests that the distribution of asset values is symmetrical and centered around the mean; which investors can make more informed decisions about their investments. Finally theorems were developed and proved show casing the progress of time varying investments and to the best of our knowledge this is the first of its kind to have assessed asset values with and without delay parameters to suit financial market for capital investments which is informative to investors in time varying investments.

**Keywords: Stock Prices, Financial markets, Stochastic analysis , Drift, volatility and Delay parameter**

**1.1 Introduction**

Stock price fluctuations create fear and as a result, people resort to criminality so as to meet individual needs, and snowballs to buying goods out of anxiety. in fact, financial analysts who invest in financial market are typically clueless of the behaviour of stock market; hence they go through this stock trading problem, and also faces the challenge of not knowing the type of stocks to be bought and sold for profit maximization. Hence relevant information on regular basis are required or needed by both financial analysis and potential investors for the prediction of stock price behaviour. The unstable characteristic and other significant influences like liquidity on stock returns, because the abrupt changes in share prices is erratic and happens regularly. So, in order to help investors and owners of corporations take decisions on the level of their investment in stock market [1], researchers are curious and fascinated in studying the behaviour of the unstable market variables.

Though, the price evolution of risky assets are generally modelled as a path or track of a risky assets that are generally of a diffusion process defined on several basic probability space, with the Geometric Browner motion , the main tool used as the established reference model, [2] Many researcher have modelled stock market prices with several ways obtained results. For example, [3] studied systems of SDEs for economic investments whose rate of returns and asset valuation follows series price index, periodic and multiplicative effects. In the research of [4] they studied the concept of asset values with delay parameter in the model and defined conditions which governed asset values as a result of delay parameter were obtained. Stochastic analysis of the behaviour of stock prices was studied by [1], and results showed that the proposed model was efficient for predicting stock prices. Similarly, [5] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in this research, the drift and volatility coefficients for the stochastic differential equations were obtained and the Euler-Maruyama method for system of SDEs was utilized to invigorate the stock prices [6], developed the geometric Brownian motion and looked at the exactness of the model with comprehensive analysis of assumed data. [7] studied the stochastic modelling of stock prices applying a method of Brownian motion model to explain the stock price time series. Yet [2] studied stochastic model of the fluctuation of stock market price. Conditions for finding out the equilibrium price, adequate conditions for robust stability and convergence to equilibrium of the growth rate of the value function of shares. However, [8] looked at a stochastic model of price changes at the floor of stock market. In the work of [9], the equilibrium price and the market growth rate of shares we determined. Past efforts for instance, [5] examined a stochastic model of several selected stocks in the Nigerian Stock Exchange (NSE) where the Euler-Maruyam method for system of (SDE) was utilized to invigorate the stock prices and result revealed that stock1 yields the best returns on investment compared to stock2, stock3 and stock4. The merit of this research over [5] is that the present research models the effect of growth-rates on stock market price estimation or forecast regarding to volatility and the drift. in fact, lots of authors has written extensively on stock market behaviors such as, [10-14] and [15-22] respectively.

However, the aim of this paper is to develop stochastic models for capital asset pricing with and without delay parameters. It is obvious that investors are really affected by their personal decisions due to expected rate of returns in their investments. This concerned the scholars of this paper to cultivate a good practical approach that can stand in terms of decisions making. It is reasonable that [3] has studied systems of SDEs for economic investments whose rate of returns and asset valuation follows series price index, periodic and multiplicative effects; whereas [4] looked at the concept of asset values with delay parameter in the model and defined conditions which governed asset values. The improvement of this paper over [3] and [4] respectively is that, the impact of volatility and delays on the values of assets, offers a precious understanding of the actions of financial markets so that the investors are able to make more informed decisions.Our innovative approach used in this study, especially via Ito’s method, extends the understanding of the dynamics of an asset in uncertainty. In addition, the empirical findings and theorems stipulated present a significant framework for further investigation, which may direct future progress in financial modelling and investment strategies. The novel idea in this paper compliments previous efforts and extends frontiers in this dynamic area of mathematical finance.

This paper is set as follows: Section 2.1 presents mathematical formulation, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

**2.1 Mathematical Formulation**

In the forgoing we state the following theorem which will help solve our problems

**Theorem 1.1:(Ito’s lemma). Let ** be a twice continuous differential function on  and let  denotes an Ito’s process.

 ,

Applying Taylor series expansion of  gives:

 ,

So, ignoring h.o.t and substituting for  we obtain







More so, given the variable  denotes stock price, then following GBM implies , the function  ,Ito’s lemma gives:

 [16] , [17], [24-25].

On the other hand, the stochastic analysis on the variations stock drift and it effects in financial markets is measured. The volatility dynamics and other drift coefficients of stock prices was taken to be constant throughout the trading days. The initial stock price which is assumed to follow diverse trend series was categorized the entire origin of stock dynamics is found in a complete probability space with a finite time investment horizon. Also in real life situations there must be delay in every circumstances. Incorporating delays in cash flows or other events, investors and analysts can set more realistic expectations about the future value of an asset, which can help prevent over or under-valuations ,hence we have the following modified system of stochastic differential equations representing different rate of returns below;

 (1.1)

 (1.2)

 (1.3)

where is an expected rate of returns on stock, is the volatility of the stock , is the relative change in the price during the period of time and  is a Wiener process,  are constants and  is periodic events parameter measuring levels of return rate .

**2.2 Method of Solution**

The propose model (1.1) - (1.3) consist of a system of variable coefficient problem of stochastic differential equations whose solutions are not trivial. we solve equations independently as follows using Ito’s theorem 1.1:

From (1.1) let 

Taking the partial derivative yields

 (1.4)

According to Ito’s gives:

 (1.5)

Subtitling (1.4) into (1.5) gives

 (1.6)

 

Integrating the above expression

 (1.7)

 

Taking ln of the both sides gives

 (1.8)

From (1.2) let 

Taking the partial derivative yields

 (1.9)

According to Ito’s gives:

 (1.10)

Substituting (1.9) into (1.10) gives

 (1.11)

 

Integrating the above expression

 (1.12)

 

Taking ln of the both sides gives

 (1.13)

From (1.3) let 

Taking the partial derivative yields

 (1.14)

According to Ito’s gives:

 (1.15)

Substituting (1.14) into (1.15) gives

 (1.16)

 

Integrating the above expression

 (1.17)

 

Taking ln of the both sides gives

 (1.18)

Incorporating a delay parameter into the solutions(1.8),(1.13) and (1.18) provides more accurate representation of the dynamics of an asset’s value over time ,as it accounts for the time-varying nature of cash flows and other factors , this gives the following results:

 (1.19)

 (1.20)

 (1.21)

where is the delay parameter of the asset values.

However, we can now generalize our modified systems to be Markovian processes, hence we have:

**Theorem 1.2 : (Markovian processes).** Let a real valued process  which is found in some filtered probability space and hence satisfies the stock dynamics below:

 for some adapted, real-valued and uniformly bounded processes  and  .By substitution  =

Now, there exists a weak solution  which has one-dimensional distribution and 

***Proof***

**Applying Ito’s**  theorem for any  . We have the following:

 

To define as follows: through



where 

Following theorem 1.2 of [23] there exist a weak solution with  That is with same one-dimensional distribution as 

**Theorem 1.3** : Let the initial stock prices of this corporate investors be  which is a nonnegative, integrable random variable with strictly increasing, continuous cumulative density function (cdf) is given as;



***Proof.***

Let be the cdf of  . Then

So to establish the claim we need to show that



Since the mapping

 is differentiable, with strickly increasing , continuous derivative



it reaches its unique minimum at 

**3 .1 Results and Discussion**

This Section presents the table results for whose solutions are in (1.8), (1.13) , (1.18) and graphical solutions (1.19-1.21) respectively. Parameter values were choosing on the basis of reflecting the actual characteristics of the asset to ensure the valuation model is realistic and captures the true value of the asset. On the aspect of delay parameter  were considered on the basis of a shorter delay may be more conservative, while a longer delay may be more aggressive, so the choice of  can represent a balance between these two extremes. Hence the following parameter values were used in the simulation study:



**Table 1: The result of volatility on the assessment of asset values when time is fixed**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Volatility |  |  |  |
| 60.77 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 60.17  53.36  45.48  37.23 | 64.53  62.62  51.78  44.02 | 4.36  9.26  6.3  6.79 |
| 50.25 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 50.76  45.02  38.36  30.78 | 53.36  48.64  42.82  36.40 | 2.6  3.62  4.46  5.62 |
| 40.10 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 39.70  35.21  30.01  24.57 | 42.58  38.82  34.17  29.05 | 2.88  3.61  4.17  4.48 |



**Table 2: The result of volatility on the assessment of asset values when time is fixed.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Volatility |  |  |  |
| 60.77 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 392.40  348.03  296.58  242.81 | 102.50  94.36  83.06  70.60 | 288.9  253.67  213.52  172.21 |
| 50.25 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 324.47  287.78  245.24  200.78 | 85.59  78.02  68.68  58.38 | 238.88  209.76  176.56  142.4 |
| 40.10 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 258.93  229.65  195.70  160.22 | 68.30  62.26  54.81  46.59 | 190.63  167.31  140.89  113.63 |



**Table 3: The result of volatility on the assessment of asset values when time is fixed**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Volatility |  |  |  |
| 60.77 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 388.50  344.57  293.62  240.40 | 103.24  94.12  82.86  70.43 | 285.26  250.45  210.76  169.97 |
| 50.25 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 321.25  284.92  242.79  198.78 | 85.37  77.83  68.51  58.24 | 235.88  207.09  45.72  140.54 |
| 40.10 | 4.0000  4.0000  4.0000  4.0000 | 0.5  0.6  0.7  0.8 | 256.36  227.37  193.75  158.63 | 68.13  62.11  54.67  46.47 | 188.23  165.26  139.08  112.16 |

From Tables 1,2 and 3 shows the impact of a delay parameter into a model increases the value of an asset in the timing of cash flows, such as a delay in receiving revenue or delay in making payments this can have a significant impact on the present value of the asset. More so, market sentiment in the that a delay parameter may represent uncertainty or hesitation in the market.

The difference between two asset values can represent the return on investment, which is the profit or loss that an investor earns from an investment over a specified period of time. Also, the different between two asset values can represent the opportunity cost, which is the cost of choosing one investment over another, see column 6 of Tables 1,2 and 3.

However, Tables 1, 2 and 3: shows increase in volatility decreases the value of asset; this means that the asset is more likely to experience large price swings in either direction. When this happens, the asset’s value can be said to have decreased because its price is more uncertain. This makes the asset less desirable for investors, who typically prefer assets that have stable, predictable prices. As a result, increased volatility can lead to a decline in the value of an asset.

In comparing asset values with and without a delay parameter: the delay in cash flows , investors may apply a higher discount rate to the delayed cash flows, which can also reduce their present value; which implies asset values that incorporate a delay parameter are more realistic than those that do not for a few reasons.



**Figure 1: Normal probability plot of Delayed asset values for first corporate investor when time is fixed with variations of volatility**



**Figure 2: Normal probability plot of Delayed asset values for second corporate investor when time is fixed with variations of volatility**



**Figure 3: Normal probability plot of Delayed asset values for third corporate investor when time is fixed with variations of volatility**

Figure 1,2 and 3 shows positive linear trend which suggests that the distribution of asset values is symmetrical and centered around the mean, which is a characteristics of a normal distribution. Also the asset values are increasing over time thus may indicate stable growth or consistent returns. The information provided by the normal probability plot can help investors make more informed decisions about their investments, which can lead to better financial outcomes.

**4.1 Conclusion**

This paper considered stochastic systems with importance on disparities of stock quantities by implementing the Ito’s method of solution where precise measures were given on the assessments of asset values with and without delay parameters. Consequently, the impressions on Tables for investors in financial markets were analyzed to establish empirically the characteristics of asset values with and without delay parameters when volatility increases. From the stochastic analysis of the problem we presume that ; increase in volatility decreases the value of assets, incorporating delay parameter increases the value of assets, the asset values with delay is realistic than those that do not for a few reasons; More so, the probability normal plot shows positive linear trend which suggests that the distribution of asset values is symmetrical and centered around the mean; which investors can make more informed decisions about their investments. Finally theorems were developed and proved show casing the progress of time varying investments. Consequently, we recommend stability analysis on stochastic differential equations with control studies in the assessment of stock variables in the next study.

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