**Stochastic Models on Variation of Capital Market Prices for Economic Investments: A comprehensive study**

**ABSTRACT**

*The study investigated stochastic models of capital market price variation for economic investments.* Money is heavily invested in time-varying assets so that the rate of return increases significantly, offsetting any unanticipated costs associated with trading activities. *The management of assets and obligations, which are essential instruments for achieving financial autonomy, is where the benefits of financial investments are found. Second-Order Differential System for variations of stock market volatility, equations and other popular models are widely recognized. This dissertation examined mathematical models that could be applied to investment plan decision-making. Stochastic systems were specifically created to take stock market price fluctuations into account. Volatility, strike price, interest rate, asset value, asset maturity time, and growth rate of the underlying asset price were the stochastic factors that were employed. The Laplace transform method, which provided exact conditions to determine the asset price function of independent investors, was used to adopt the analytical answers. Furthermore, in order to examine the share prices of Fidelity and Access banks, respectively, the study introduced probability transition matrices and investigated the characteristics of fundamental matrix solutions. Though modified Bessel solutions won't crash, crashes occurred for analytical solutions that used the Bessel function, which led to panic buying and a drop in asset value.*

**Keywords: Laplace transform, differential equations, stochastic differential equations, Bessel functions**

**INTRODUCTION**

Generally speaking, trying to turn a profit in real life is focused on trading and investing. The majority of participants in the financial markets are traders and investors because it's how they generate income from trading and investing. All things considered, purchasing and holding are how investors deal with short-term and long-term investment goals. While traders join and leave positions quickly, taking smaller, more frequent profits, they also take advantage of rising and declining markets. Consequently, the advantage of investing is in its ability to progressively accumulate wealth over an extended duration by means of purchasing and maintaining investment portfolios. However, creating both short- and long-term business plans requires financial resources, which gradually increases investor returns. The foundation for an individual or organization to create riches and lead a pleasant life is financial assets. Money is heavily invested in time-varying assets so that the rate of return increases significantly, offsetting any unanticipated costs associated with trading activities. Furthermore, an investor may experience a cash flow crisis due to unanticipated events like a market crash, a flood, youth restiveness or a political crisis that could have an impact on their assets directly or indirectly. It is observed that an investor will select one portfolio rather than another based on the criterion that the probability of the portfolio's return falling below a minimum desired threshold is minimized. According to Penrose and Penrose (2019), this theory assumes that the investors are rational and the market is efficient and perfect. This means that the investors are capable of making economic decisionsout of reason as opposed to intuition. Thus, they know when to invest or not to invest and that they can predict the changes in the market (Kliber & Rutkowska-Ziarko, 2021; Kennedy & Gilbert, 2023).

Nonetheless, the general public, in addition to mathematicians and investors, places a high value on mathematical analysis in finance. Therefore, comprehending its dynamics will aid government officials, opinion leaders, and economists in properly planning their investments for the future. Therefore, the study's particular field of interest is financial mathematics, which deals with the emergence and occurrences of financial issues. Furthermore, it is quite vital to comprehend the dynamic character of the financial market's influence. It is crucial from a practical standpoint to be able to comprehend financial factors, their dynamic interactions, and how traders and investors are affected. In order to generate realistic stock amounts or variables, it is also necessary to analyze the value of stock return rates and other pertinent stochastic factors in the model. These assessments require well-articulated, precisely formulated, and analytical answers. In order to incorporate additional environmental impacts, solving such issues requires a model consisting of a set of differential equations with stochastic parameters. As a result, the analytical solution is chosen based on the particulars of the problem under consideration. A differential equation is a crucial tool for combining various parts into a single system and examining the interactions that occur between these parts, some of which may continue to be independent of one another. Davies (2005), (2006).

Numerous factors influence the value and daily price of stocks inside the financial sector; these factors additionally specify the kind of market's volatility. As observed in previous studies, the stock market is highly volatile and very unstable which needs to be examined appropriately to enable investors, economists and owners of corporations should base important choices on the size of their investments. A higher level of product market competition and a higher level of policy uncertainty increase the positive effect of industry cash flow volatility on stock price crash risks, which confirms the moderating effect of product market competition and policy uncertainty (Wang et al. 2022).  A lot of scholars have solved 2nd order differential equations using the Frobenius method in assessing asset values of investments without considering the validity of the key parameters of the Black-Scholes equation. Hence, we shall apply the method of Laplace transform to the differential equation system in an effort to realistically evaluate asset values in time-varying investments based on analytical solutions Also, the fundamental matrix approach to evaluating the impact of time and volatility in share prices will also help investors and owners of corporation in their financial decision making. This forms the novelties of this study as it will deepen the domain in which such problems can be applied.

A modified Ordinary Differential Equation (ODE) method was developed by Mihova et al. (2022) and used to forecast the stock values of around four Bulgarian enterprises. Using quantitative data on daily closing share prices for 2020–2022, they developed models for predicting the price of stocks. After calculating and analyzing each of the observed stocks using the best models, they looked at the expected rates of return and the variances of those rates of return, respectively.

Anokye et al. (2019) employed characteristic equation approaches to study the dynamics of equilibrium pricing with differential and delay differential equations. They used roots of characteristic equilibrium approaches to examine the quantitative behaviour of the differential and delay differential models in relation to changes in equilibrium pricing. Their research showed that the time delay factors in the supply function of price lead current prices in the delay differential model to be unexpected at initially. On the other hand, current prices in differential models are anticipated instantly because they automatically converge to equilibrium price points that are determined by the system.

Amadi et al. (2022) solved differential equations and stochastic differential equations of time-varying investment returns using multiplicative and multiplicative inverse trend series. They discovered correct conditions governing the asset price return rate. The proposed model proved, in both deterministic and stochastic systems, that a multiplicative inverse trend series outperformed a multiplicative trend in terms of effectiveness and reliability.

Osu (2010) established the necessary conditions for dynamic stability and the convergence to equilibrium of the growth rate of the value function (output) of stock shares using a stochastic model of stock market price volatility. He also managed to pin down the exact parameters that would allow him to calculate the equilibrium price. The volatility-filled drift parameters of the pricing process operate as the model's constraint.

Amadi and Charles (2022) investigated the solution of stochastic differential equations and differential equations including time-varying investment returns. They achieved this by analyzing the asset price return rate using multiplicative and multiplicative inverse trend series. The results demonstrate that multiplicative inverse trend series outperform multiplicative trends in terms of efficiency and reliability in both stochastic and deterministic systems.

Okpoye et al. (2023) relied extensively on multiplicative, additive, and additive inverse effects to establish the stock rate of returns using stochastic analysis on the volatility of asset value functions. This proved conclusively that the analytical answer was correct by using real stock prices.

In 2022, Amadi and Wobo came out with a mathematical model analysis to predict changes in stock market values. For the purpose of choosing the optimal model, they investigate several approaches to parameter estimation for the Weibull distribution and employ mean square error (MSE). By looking at the features of the basic matrix solution, from which they were able to get one year's worth of anticipated stock prices and asset returns, the estimated findings were rationally extended to generate a matrix that will be effective in predicting other commodity price processes. Building on the foundational metric system, they demonstrated many tiers of changes affecting stock markets both immediately and in the long run. As a result, the fundamental matrix is a powerful tool for gauging how stock prices will move in the future.

**Problem Formulation on Price of Asset Equations**

The price of assets which grow explicitly needs to be examined due to key stock market quantities namely: strike price, maturity time of the underlying assets, interest rates stock price and prices etc. These parameters are affected due to some environmental effects. Let  and  first, second, third and fourth corporate investors. Hence, we assume portfolios of investment by reformulating the work of Amadi et al. (2022) as follows:

** (**3.1)

 (3.2)

 (3.3)

 (3.4)

With the following boundary conditions

****

 (3.5)

Where represents volatility, is the strike price, is the interest rate, is the price of worth of assets, represents maturity time of assets,  is a constant and  is the growth rate of the underlying price of asset.

**Method of Solution**

The Laplace transform for second-order differential equation was adopted.

Hence, solving the homogenous part which can be written as,

****  (3.6)

Taking the Laplace transform of (3.11) gives

 (3.7)

 (3.8)

To clear the bracket above yields

 (3.9)

 (3.10)

 (3.11)

By factorization, (3.11) gives

 (3.12)

 (3.13)

(3.13) becomes a differential equation. Solving using variable separable, (3.14)

 (3.15)

Integrating both sides of (3.15) gives

 (3.16)

 (3.17)

Taking natural log of both sides,

 (3.18)

Recall that , this implies

 (3.19)

Taking the Laplace inverse of (3.19),

 (3.20)

Hence the inverse Laplace transform gives,

 (3.21)

(3.21) becomes the Modified Bessel function of order zero because of its imaginary part.

 (3.22)

Using (3.20) in (3.21) yields

 (3.23)

To obtain the non-homogenous part of (3.1), we assume a solution for particular integral. Let the particular integral be;

 (3.24)

Differentiating (3.24) up to 2nd order,

 (3.25)

 (3.26)

Substituting (3.24), (3.25) and (3.26) into (3.1) gives

 (3.27)

 (3.28)

Because (3.1) and (3.2) are related, the solution to (3.1) depends on the solution to (3.2), making the right-hand side of equation (3.1) a linked differential equation. In the RHS of (3.1), we replace the solution of (3.2), which suggests, (3.29)

Combining the terms above gives,

 (3.30)

Putting (3.30) into (3.29) gives

 (3.31)

Putting (3.23) and (3.31) into (3.28) gives,

 (3.32)

Applying the boundary conditions in (3.5) gives

 (3.33)

Solving for  in (3.33) gives

 (3.34)

Making the subject



 (3.35)

Substituting (3.35) into (3.33) gives,

 (3.36)

Combining (3.36) and (3.32)



 (3.37)

Where,



From (3.2), taking the Laplace transform gives

 (3.38)

 (3.39)

To clear the bracket above yields

 (3.40)

 (3.41)

 (3.42)

 (3.43)

 (3.44)

(3.44) becomes a differential equation. Solving using variably separable,

 (3.45)

 (3.46)

Integrating both sides of (3.46) gives

 (3.47)

 (3.48)

Taking natural log of both sides,

 (3.49)

Recall that , this implies

 (3.50)

Taking the Laplace inverse of (3.50),

 (3.51)

Hence the inverse Laplace transform gives,

 (3.52)

Using the boundary condition of (3.5) in solving (3.52),

 (3.53)

Putting (3.53) into (3.52) gives,

 (3.54)

Where (3.54) is a Bessel function of order zero given by,



From (3.3), taking Laplace transform gives

 (3.55)

 (3.56)

To clear the bracket above yields

 (3.57) (3.58) (3.59)

 (3.60)

 (3.61)

(3.61) becomes a differential equation. Solving using variably separable,

 (3.62)

 (3.63)

Integrating both sides of (3.63) gives

 (3.64)

 (3.65)

Taking a natural log of both sides,

 (3.66)

Recall that , this implies

 (3.67)

Taking the Laplace inverse of (3.67),

 (3.68)

Hence the inverse Laplace transform gives,

 (3.69)

Using the boundary condition of (3.5) in solving (3.69) by substitution,

 (3.70)

Putting (3.70) into (3.69) gives,

 (3.71)

Where  is the Modified Bessel function of order zero.

From (3.4), taking Laplace transform gives

 (3.72)

 (3.73)

To clear the bracket above yields

 (3.74)

 (3.75)

 (3.76)

 (3.77)

 (3.78)

(3.78) becomes a differential equation. Solving using variably separable,

 (3.79)

 (3.80)

Integrating both sides of (3.80) gives

 (3.81)

 (3.82)

Taking a natural log of both sides,

 (3.83)

Recall that , this implies

 (3.84)

Taking the Laplace inverse of (3.84),

 (3.85)

Hence the inverse Laplace transform gives,

 (3.86)

Using the boundary condition of (3.5) in solving (3.86),

 (3.87)

Putting (3.87) into (3.86) gives,

 (3.88)

Where is the Bessel function of order zero.

**Symmetric Characteristics of the Models**

Due to its symmetric property, the price asset solution has a normal distribution. In order to determine the symmetric properties of assets, the solution will therefore be analyzed. Thus, the following is how we express the theorem:

**Theorem 1(Symmetric properties):** The answers (3.54), (3.71) and (3.88) are symmetrical about the curve. That is, 

**Proof**

From (3.54)







Differentiating  with respect to 







From (3.71)







Differentiating  with respect to 







From (3.88)







Differentiating  with respect to 







**RESULT**



Where, 



Where 



Where, 



Where 

**RESULTS and DISCUSSION**

Equation (3.42) illustrates how the capital market fluctuates in reality as a sequence of business (investment) cycles centred on a secular trend. In the event that a crash is anticipated in the future, the bubble will not begin. As a result, the efficient market theory holds that bubbles are impossible. This agreed with Osu (2010).

Equation (3.59) shows that  is a result of asand as the value of an independent investor's output (or a firm's portfolio) goes negative, it indicates that the capital market has crashed. A crash is defined as a sharp decline in the market's overall value, which leads to a situation when most investors attempt to exit the market at the same moment and suffer enormous losses. During a crash, investors panic sell in an effort to transfer their dropping shares to other investors in an effort to limit further losses. Panic selling fuels the market's decline, which ultimately leads to a crash that impacts everyone. When stock market crashes are followed by depressions, investors' shares are worth far less than what they originally invested. This also applies to equation (3.93).

A bubble, or a sudden increase in the value of an item or group of resources in an ongoing procedure, has just occurred because, according to equation (3.76), is a function of and  increases without bounds. The first ascent creates anticipation of more increases and draws in new buyers. Speculators are typically more interested in trading profits than using the asset as a source of income.

**CONCLUSION**

In this study, Second Order System of Differential Equations using the Laplace transform was considered which comprised of the following stochastic parameters; volatility, interest rate, maturity time of investments and strike price which are contributors to the overall price of the first, and second, third and fourth corporate investors. Accordingly, the findings indicate that:

1. crashes will happen for analytical solutions using the Bessel function, resulting in panic purchasing and a decline in asset value
2. modified Bessel solutions won't experience any crashes.

Finally, solving the stochastic models with variable parameters will be of great interest.

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1.

2.

3.

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