**Stochastic Analysis on Share Price Movements with Time-Delay in Finite State**

**Abstract**

 Share prices changes over time and can be viewed as a stochastic process. In this paper, stochastic analysis of Markov chain in the closing share price data of Access and Fidelity (2016-2022) is considered. The share prices were transformed into 3-steps transition probability matrix solution to cover this number of years. From the solution matrix of stochastic analysis showed that Access bank, PLC has the best probability of price increasing in the near future: 12%, best probability of reducing in future by 21% and best probability of no-change in the near future by 20% which is a tool for proper decision making in the day-to-day management of the bank. More so, Fidelity bank, PLC has the best probability of price increasing in the near future: 10%, best probability of reducing in future by 23% and best probability of no-change in the near future by 22% which is also a tool for proper decision making in the day-to-day management of the bank. Finally from the two banks merged; has the best probability of price increasing in the near future: 12%, best probability of reducing in future by 67% and best probability of no-change in the near future by 21% , the study introduced the concept of time dependent delay parameter in the share price movements in finite state which shows lots of increase in the share price of each banks. The future share price percentage changes were known; which is a tool for proper decision making in the day-to-day management of the banks, hence the two banks are profit making organization and are hopeful for future investment plans both short or long term respectively.

**Keywords: Share price, Markov Chain, Access and Fidelity, Stochastic analysis and Transition matrix.**

**1 Introduction**

“It is understandable that stock price changes over long or short trading period of time is seen as a stochastic formation which can be modeled for the purpose of empirical findings. Since the most prominent features of financial market in Nigeria are increasing and decreasing eminence of share prices. The Nigerian stock Exchange (NSE) plays vital roles in raising capital funds and also acts as a medium between firms and the investors. Therefore, empirical studies of NSE show that results can be obtained when the stock price dynamics are well known”, [1]. “Hence stock market performance and its processes have popularity as a significant viable investment field within financial market. Many scholars has widely written on stock market price formations such as” [2]-[8] etc. All the same, the price evolution of a risky assets are usually modeled as the trajectory of a risky assets that are usually of a Markov process defined on some underlying transition probability state space . From the stochastic point of view the method of Markov chain stipulates a system of transition matrix of an element beginning from one state to another; ascertaining the transition as a random process, show-casing the memory-less property of Markov chain. That is to say that the future state of any process strictly depends on its current state but not its past series of ideas acquired over time. Markov chain is one of the most well developed theories of stochastic process with its applications in growing field of science and technology.

Though, a lots scholar has written widely on the modeling stock price using Markov chain and results obtained in various ways. For instance,[9] “considered stochastic analysis of share prices. Results showed precise condition of determining expected mean return time for stock price; improving investment decision based on highest transition probabilities. In the same manner”, [1] “examined stock market prices due to its fluctuations and influences in financial lives and economic health of a country. Their findings showed that stock price is a random work and no investor can alter the fairness and unfairness of a stock price as defined by expectation”.

However , in the works of [10] studied “the behavior of stock market using Markov chain. The study reveals that regardless of bank’s current share price steady state probabilities of share price remain the same all through the iteration”. [11] “Introduced a Markov chain model for stock market trend forecasting. The study revealed the Markov chain model was more effective to analyze and predict the stock market index and closing stock price under the market mechanism”. [12] “Studied long run prospects of security prices in Nigeria where the data were collected from the randomly selected banks from the banking sector of Nigeria. The analysis suggested that the price level of Nigerian bank were likely to remain relatively stable in the long run irrespective of the current situations”.[13] “examined the long run behavior of the closing price of shares of eight Nigerian banks using Markov chain model. They computed limiting distribution transition probability matrix of the of share price and found that despite of the current situation in the market there is hope for Nigerian bank stocks. It was concluded that the results derive from the study will be useful to investors”.

"In this paper, stochastic analysis of Markov chain in the closing share price data of Access and Fidelity (2016-2022) is considered. The share prices were transformed into 3-steps transition probability matrix solution to cover this number of years. The study introduced the concept of time dependent delay parameter in the share price movements in finite state which shows lots of increase in the share price of each banks. The future share price percentage changes were known; which is a tool for proper decision making in the day-to-day management of the banks, hence the two banks are profit making organization and are hopeful for future investment plans both short or long term respectively”.[15]

The aim of this paper is first, to present the share prices of Access, Fidelity and their future merging in finite state using Markov chain which is aimed at determining the impact of time delay in share price movements as it affects the share prices of two banks under-study.

This paper is arranged as follows: Section 2.1 presents material and methods; results and discussion are presented in Section 3.1 and the paper is concluded in Section 4.1.

**2.1 Material and Methods**

For the purpose of understanding this paper on Markov chain we start from defining stochastic process. Stochastic process can be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

**Definition 1:**  A stochastic process  is said to be a Markov chain if Markov property is satisfied :

  (1)

For all ** .**

It is sufficient to know that the Markov property given (1) is equivalent to easy of the following for each  . 

  (2)

 

Assuming  implies that the chain is in the  state at the  step.it can also be said that the chain’ having the value i’ or ‘ being in state i’. The idea behind the chain is described by its transition probabilities:

  (3)

They are dependent on ****

**Definition 2:** The chain is said to be homogeneous if the following are stated below

  (4)

For all ****

The transition matrix  is  matrix of transition probabilities.

  (5)

Hence, the transition probabilities with homogenous Markov chain are always stationary at a point.

**Theorem 3:** Suppose  is a stochastic matrix which implies the following:

i) has non-negative entries or  (ii)

which is stationarity or point of convergence.

Proof:( i) each associated entry in  is a transition probability and being probability .

(ii) 

Which is stationarity.



**Theorem 4 :( Chapman-Kolmogorov Equations).**

**** Since and so on the  power of  .

Proof: 

Using the following in probability rule:

 and setting 

Using Markov property yields



To obtain an estimates of the transition probability as follows

 

 

where is the number of states.

  (6)

**3.2 Developing Markov Chain Model for Stochastic Analysis of Access, Fidelity Share Price and their future merging.**

For proper accuracy of Markov chain model for future events; it needs to be developed for prediction of share price movement. The initial share prices needs to be in three finite states as follows:

**R:** represents the probability of share price reducing in near future

**I** : represents the probability of share price increasing in near future

**NO-change** : represents the probability of share price not changing in near future

However, probability of transition matrix shows the proper explanation of Markov chain. Every element in the matrix communicates. In order to form three states of Markov process we need to have the following table below:

**Chart 1: Transition Probability Matrix**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **State** | **1** | **2** | **3** | **Total of Row** |
| **1** |  |  |  |  |
| **2** |  |  |  |  |
| **3** |  |  |  |  |

However, for  is an estimate of the transition matrix.

  (7)

  (8)

  (9)

Setting 

Introducing time dependent delay in (7-9) gives the following:

However, for  is an estimate of the transition matrix.

  (10)

  (11)

Where  represents time and  represents delay parameter.

  (12)

**3.1 Results and Discussion**

To illustrate the share price of two banks movement with time dependent delay in finite states using Markov chain model extracted from [14] were used for the study

**Probability Transition Matrix For Access Bank Share Price**

 

**Access bank (2016-2022)**: Shows 67% probability of reducing its price in near future; 13% chance of increasing its price in the near future; 20% chance of no change in price. Also in the same situations 29% chance of reducing its price; 36% chance of increasing its price and 34% chance of no change in price. Lastly 21% chance of reducing its price 15% chance of increasing its price and 63% Chance of no change in price. The above valuations provides an eye opener to the management of Access bank, PLC that will enhance investment decisions.

**Effect Of Delay On Its Transition Matrix Of Access Bank Share Price**



**The effects of delay on the share price movements of Access bank (2016-2022)**: Shows 383% probability of reducing its price in near future; 75% chance of increasing its price in the near future; 118% chance of no change in price. Also in the same situations 169% chance of reducing its price; 209% chance of increasing its price and 197% chance of no change in price. Lastly 123% chance of reducing its price 88% chance of increasing its price and 365% Chance of no change in price. The above valuations provide an eye opener to the management of Access bank, PLC that will enhance investment decisions.

**Probability Transition Matrix For Fidelity Bank Share Price**

 

**Fidelity (2016-2022)**: Expressed 67% of reducing its price; 10% chance of increasing its price in the near future; 22% chance of no change in price. Moreover in the same settings 23% chance of reducing its price; 46% chance of increasing its price and 31% chance of no change in price. Finally 23% chance of reducing its price 13% chance of increasing its price and 64% Chance of no change in price. In the stochastic analysis assessments of Fidelity share prices provides future movements of share prices that will enhance their investment decision making in the long run.

**Effect Of Time Delay On Transition Matrix Fidelity Bank Share Price**

 

**The effects of delay on the share price movements of Fidelity (2016-2022)**: Expressed 388% of reducing its price; 58% chance of increasing its price in the near future; 129% chance of no change in price. Moreover in the same settings 133% chance of reducing its price; 265% chance of increasing its price and 177% chance of no change in price. Finally 133% chance of reducing its price 76% chance of increasing its price and 366% Chance of no change in price. In the stochastic analysis assessments of Fidelity share prices provides future movements of share prices that will enhance their investment decision making in the long run.

 **Transition Matrix Of The Two Bank Merged according to [14].**

 

**The two banks merged: Access and Fidelity (2016-2022)**: Represents 67% of reducing its price; 12% chance of increasing its price in the near future; 21% chance of no change in price. Also in the same state of affairs 26% chance of reducing its price; 41% chance of increasing its price and 33% chance of no change in price. In conclusion 22% chance of reducing its price 14% chance of increasing its price and 64% Chance of no change in price. In all, the overall predicted prices for the above companies gives: 22% chance of reducing its price, 14% chance of increasing its price and 64% chance of no change in price. The entire entry specifies price changes for short and long term investment plans of Access and Fidelity banks.

**Effect Of Time Delay On The Transition Matrix Of The Two Banks Merged**

 

**The effects of delay on the share price movements of two banks merged: Access and Fidelity (2016-2022)**: Represents 385% of reducing its price; 66% chance of increasing its price in the near future; 123% chance of no change in price. Also in the same state of affairs 151% chance of reducing its price; 237% chance of increasing its price and 187% chance of no change in price. In conclusion 128% chance of reducing its price 82% chance of increasing its price and 365% Chance of no change in price. The entire entry specifies price changes for short and long term investment plans of Access and Fidelity banks.

However, delay parameter has good influence on the price movements of Access, Fidelity and Acc-Fid banks. It implies the movement of share price is time dependent hence they are profit maximizing. This situation informs the management of the two banks to delay before their shares can be sold in order to record huge profit margin.

**Table 1: The Percentage Price Changes Of Future Movement Of Access Bank Share Price**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **0.6656** | **0.1299** | **0.2045** |
| 0.20.250.3 | 0.13310.16640.1997 | 0.02590.03250.0389 | 0.04090.05110.0614 |
|  | **0.2937** | **0.3643** | **0.3420** |
| 0.20.250.3 | 0.05870.07340.0881 | 0.07290.09110.1093 | 0.06840.68550.1026 |
|  | **0.2131** | **0.1527** | **0.6342** |
| 0.20.250.3 | 0.04260.05330.0639 | 0.03050.03820.0458 | 0.12680.15860.1903 |

**Table 2: The Percentage Price Changes Of Future Movement Of Fidelity Bank Share Prices**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **0.6748** | **0.1008** | **0.2244** |
| 0.20.250.3 | 0.13490.16870.2024 | 0.02020.02520.0302 | 0.04490.05610.0673 |
|  | **0.2319** | **0.4601** | **0.3080** |
| 0.20.250.3 | 0.04640.05790.0696 | 0.09200.11500.1380 | 0.06160.07700.0924 |
|  | **0.2305** | **0.1327** | **0.6368** |
| 0.20.250.3 | 0.04610.05760.0692 | 0.02650.03320.0398 | 0.12740.15920.1910 |

**Table 3: The Percentage Price Changes Of Future Movement Of The Two Banks Merged**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **0.6702** | **0.1154** | **0.2145** |
| 0.20.250.3 | 0.13400.16760.2011 | 0.02310.02890.0346 | 0.04290.05360.0644 |
|  | **0.2632** | **0.4117** | **0.3252** |
| 0.20.250.3 | 0.05260.06580.0789 | 0.08230.10290.1235 | 0.06500.08130.0976 |
|  | **0.2219** | **0.1426** | **0.6355** |
| 0.20.250.3 | 0.04440.05550.0666 | 0.02850.03570.0428 | 0.12710.15890.1907 |

Tables’ 1-3 displays the level of percentage changes in the share price movements of two banks even as they merged in future. It can be observed that increase in the percentage changes increases each share price entry in the movement. Critical looking at the share price changes will observe that the change in percentage is informative; to the management of the two banks for the purpose of decision making.

**4.1. Conclusion**

 This paper studied stochastic analysis of Markov chain in the closing share price data of Access and Fidelity (2016-2022). The share prices were transformed into 3-steps transition probability matrix solution to cover this number of years. From the solution matrix of stochastic analysis showed that Access bank, PLC has the best probability of price increasing in the near future: 12%, best probability of reducing in future by 21% and best probability of no-change in the near future by 20% which is a tool for proper decision making in the day-to-day management of the bank. More so, Fidelity bank, PLC has the best probability of price increasing in the near future: 10%, best probability of reducing in future by 23% and best probability of no-change in the near future by 22% which is also a tool for proper decision making in the day-to-day management of the bank. Finally from the two banks merged; has the best probability of price increasing in the near future: 12%, best probability of reducing in future by 67% and best probability of no-change in the near future by 21% , the study introduced the concept of time dependent parameter in the share price movements in finite state which shows lots of increase in the share price of each banks. The future share price percentage changes were known; which is a tool for proper decision making in the day-to-day management of the banks. However, incorporating principal component analysis in the share price of the two banks is suggested as an interesting area of further study.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

Option 2:

Author(s) hereby declare that generative AI technologies such as Large Language Models, etc. have been used during the writing or editing of manuscripts. This explanation will include the name, version, model, and source of the generative AI technology and as well as all input prompts provided to the generative AI technology

Details of the AI usage are given below:

1.

2.

3.

**Reference**

[1] Agwuegbo, S.ON., Adewole, A.P. and Maduegbuna, A.N(2010). A Random walk for stock Market prices. Journal of Mathematics and Statistics,6(3),342-346.

[2] Amadi, I. U, Igbudu R and Azor P. A.(2022). Stochastic Analysis of the Impact of Growth-Rates on Stock Market Prices. *Asian journal of Economic, Business and Accounting.*

[3] Adeosun,M.E., Edeki,S.O. and Ugbebor,O.O(2015).Stochastic Analysis of Stock Market Price Models: A case Study of the Nigerian Stock Exchange(NSE*). WSEAS transactions on Mathematics,14,*353-363.

[4] Davies, I. Amadi, I.U and Ndu, R.I(2019).Stability Analysis of Stochastic Model for Stock Market Prices. *International journal of Mathematics and Computational Methods ,4,*79-86.

[5] Ofomata,A.I.O.,Inyama,S.C.,Umana,R.A . and Omane,A.O(2017).A stochastic Model of the Dynamics of Stock Price for Forecasting. *Journal of Advances in Mathematics and Computer Science*.*25(6*),1-24.

 [6] Dmouj,A.(2006).Stock price modeling:Theory and practice.*BIM Paper.*

[7] Ugbebor,O.O,Onah,S.E. and Ojowo, O.(2001). An Empirical Stochastic Model of Stock Price Changes. *Journal Nigerian Mathematical Society,20*,95-101

[8] Osu, B.O,Okoroafor,A.C. and Olunkwa, C.(2009). Stability Analysis of Stochastic model of Stock market price. *African journal of Mathematics and Computer Science 2(6),98-103*.

 [9] Mettle, F.O, Quaye, E.N.B and Laryea R.A(2014). A Methodology for stochastic Analysis of share prices as Markov chains with finite States.http://www.springerplus.com/content/3/1/057.

[10] Bairagi A. and CH.Kakaty S.(2015). Analysis of stock market price behavior: A markov chain approach. *International journal of recent scientific research*. Vol .6,issue 10, 7061-7066.

[11] Zhang D. and Zhang X. (2009).Study on forecasting the stock market trend based on stochastic analysis method. *International journal of Business and management*. 4(6). 163-170.

[12] Eseoghene, J.I.(2011). The Long run propect of stocks in the Nigeria capital Market: a Markovian Analysis*. JORIND* (9)1.

[13] Christain, E.O and Timothy, K.S.(2014). On predicting the long run behavior of Nigerian Bank stock prices: a Markov chain approach. *American journal of applied mathematics and statistics,*2,4,212-215.

[14] Osu, B.O., Emenyonu, S.C ., Ogbogbo, C.P. and Olunkwa, C.(2019). Markov Models on Share Price Movements in Nigeria Stock Market Capitalization, *Applied Mathematics and Information Sciences an International Journal,*N0 2,1-9.

[15] Wokoma, D. S. A., & Amadi, I. U. (2023). The Influence of Markov Chain and Properties of Principal Component Solutions in The Analysis of Share Price Movements for Stock Market. British Journal of Multidisciplinary and Advanced Studies, 4(6), 23-35.