**Stochastic Approximation on Constant Elasticity of Variance Equations for Stock Market Prices**

**Abstract**

Applications of Constant Elasticity of Variance (CEV) Equations were considered in assessing the wealth of two different corporate investors and describing the behavior of a security’s volatility over time. The methods of Ito’s were explored and a précised condition of obtaining wealth of each corporate investor is given to illustrate the effectiveness of the systems. Each investment solutions suggest distinctive perceptions on CEV dynamics of assessing wealth of corporate investors. The simulation results presented graphically with the use of MATLAB and discussions of these graphical solutions with relevant parameters were addressed effectively all in this paper.

**Keywords: CEV, Investors, Stock Market, Wealth and Stochastic Analysis**

* 1. **Introduction**

The nature of stock prices has been unstable, seasonal, time-dependent and highly volatile and therefore unpredictable. This is mostly due to uncertainties that arise from natural calamities, global trends, socio-political policies which may have unprecedented impact on the demand and supply of stocks [1]. Because of this, investors now have to go beyond studying the company's history, performance and development prospects of such fundamentals, but also be familiar with the variety of technical analysis in order to win a huge return on investment and become a successful investor. Stock trend analysis plays an important role in practical stock trading.

 In analyzing stock trends, there are various tools which can be used depending on the factors that shape the stock price trends, and stochastic analysis include the Constant Elasticity of Variance (CEV) model which was introduced to effectively compliment one of the limitations of Stochastic Differential Equation (SDE). Cox noticed the fact that constant volatility were unrealistic; a power law relationship between the asset’s volatility and its price levels. This offers more realistic and flexible way of modeling time-varying volatility for investors or owner’s corporation. More so, this CEV model can also be twisted by incorporating mean-reverting properties into the volatility process. This gives makes it Modified Constant Elasticity of Variance (MCEV) which gives room for the model to better address short term and long term fluctuations thereby making more robust for suitable market conditions. In all, the usefulness of CEV are as follows: by the use of CEV equation, corporate investors can better understand the volatility of their investments and make more informed decisions about risk management; the CEV equation can be used to help corporate investors optimize a mix of investments with varying levels of volatility and returns and finally understanding the patterns of volatility over time, corporate investor can make more informed decisions about when to buy and sell investments.

A lot of researchers have modeled stochastic analysis of stock market prices with several ways obtained results. For example,[3] studied the unstable feature of stock market forces, making use of suggested differential equation model. In the research of [4], stability analysis of stochastic model of price change at the floor of a stock market was considered and precise conditions were obtained which determined the equilibrium price and growth rate of asset shares. Stochastic analysis of the behavior of stock prices was studied by [1], and results showed that the proposed model was efficient for predicting stock prices. Similarly, [5] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in this research, the drift and volatility coefficients for the stochastic differential equations were obtained and the Euler-Maruyama method for system of SDEs was utilized to invigorate the stock prices. In work of [6] Geometric model were used to predict stock market prices. Thus many scholars has written extensively on this subject matter such as [7-10] etc.

Meanwhile, [11] considered the option pricing implications of the CEV model in the Nigerian Stock Exchange (NSE), [12] had an empirical study of the CEV model using data from NSE and discovered that CEV model gives a reasonable representation of stock returns. [13] analysed option pricing and Greeks under the Modified Constant Elasticity of Variance(MCEV) model with stochastic volatility. Details of CEV and MCEV can be in the following papers such as [13-19] etc.

However, we considered CEV and MCEV equations in assessing two different wealth of Dangote investments and describing the behavior of a security’s volatility over time which was not considered by previous efforts. Precise conditions were given to illustrate the effectiveness of the systems. Each investment solutions suggest distinctive perception on CEV dynamics of assessing wealth of corporate investors. This paper extends the work of [9] by incorporating probability parameters in one of CEV and MCEV equations which can better balance investor’s portfolios by selecting a mix of investments with different levels of risk and returns.

The paper is arranged in the following ways: Section 2.1 Mathematical preliminaries, Section 3.1 present Results and Discussion and this paper is concluded in Section 4.1

**2.1. Mathematical Framework**

To have good understanding of this paper we define the following in which the study hinges:

**Definition 1.1: Stochastic Processes :** Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

**Definition 1.2: Constant Elasticity of Variance**: This is an equation that describes the behavior of a security’s volatility over time. It is based on the assumption that the volatility of a security’s returns is not constant, but rather follows an exponential growth pattern.

**Definition 1.3:Stochastic Differential Equation:** A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that  is a probability space with filteration  and  an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of 

 

 

where is an n-dimensional random variable and coefficient functions are in the form . SDE can also be written in the form of integral as follows:

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Where  are known as stochastic differentials. The  is a valued stochastic process.

**Theorem 1: (Ito’s lemma).** Let **** be a twice continuous differential function on  and let  denotes an Ito’s process

  ,

Applying Taylor series expansion of  gives:

  ,

So, ignoring h.o.t and substituting for  we obtain

 

 

 

More so, given the variable  denotes stock price, and hence, the function  ,Ito’s lemma gives:

 

The details of the above concepts can be seen in [18-19] respectively.

**2.1.1 Problem Formulation**

 Considering stochastic systems enforced with the dynamics of stock quantities which is said to have a complete probability space such that a finite time investment horizon.

 Hence the system of Constant Elasticity of Variance equation and its modifications are given as follows :

  (1)

  (2)

  (3)

where,  are asset prices, The expression , which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion. represents elasticity parameter,  is the price process of the stock market under modified CEV ,  and  are volatility of the stock market, represents modification factor and  represents appreciation rates.

**2.1.2 Method of Solution**

 We implement the methods of Ito’s lemma in solving for To seize this problem we note that we can forecast the future worth of the asset with sureness.

From (1) Let  so differentiating partially gives

  (4)

According to Ito’s gives

  (5)

Substituting (6) and (7) into (1) gives

 

 

Integrating both sides , talking upper and lower limits gives

  (6)

Taking the ln of the both sides

  (7)

From (2) Let  so differentiating partially gives

  (8)

According to Ito’s gives

  (9)

Substituting (8) and (9) into (2) gives

 

 

Integrating both sides , talking upper and lower limits gives

  (10)

Taking the ln of the both sides

  (11)

**3.1 Analysis and Results**

This Section presents analyzed results whose methods are stated in Section 2.1. Hence we have the following parameter values: ****which were implemented using Matlab programming software:



**Figure 1: Constant Elasticity of Variance on Assessing the Wealth of First Corporate Investor**

Figure 1 shows the relationship between the wealth and the returns is not perfectly linear , but it still has some degree of linearity. It further means that the wealth of the investor will still increase in response to the performance of the investment, but the change in wealth may not be as directly proportional to the returns as it would be a purely linear relationship.



**Figure 2: Constant Elasticity of Variance with a Probability term graphical on Assessing the Wealth of Second Corporate Investor**

As it can be seen in Figure 2, that the wealth of the investor is directly proportional to the returns of the investment. The investor’s wealth will increase in a linear or straight line, fashion in response to the performance of the investment. This can be useful for investors who are working for predictable, stable returns from their investments.



**Figure 3: Modified Constant Elasticity of Variance graphical on Assessing the Wealth of First Corporate Investor**

Here it means that the wealth is growing at an increasing rate over time, rather than at a constant rate. This could be due to factors such as compounding returns, reinvestment of profits, or a rapidly growing industry or market. Exponential growth in wealth can be extremely beneficial for investors, as it can lead to rapid accumulation of wealth over time. However, its importance to remember that exponential growth is not always sustainable, and investors should carefully consider the risks and potential returns of any investment.



**Figure 4: Modified Constant Elasticity of Variance with a Probability term on Assessing the Wealth of Third Corporate Investor**

In Figure 4 , the wealth of a corporate investor is linear but negative , it means that the wealth of the investor is decreasing at a steady rate over time, rather than increasing. This could be due to factors such as negative returns on investments, losses in the market or cost associated with running the business. A linear decrease in wealth is more manageable than an exponential decrease, as it is easier to predict and plan for. However, investors should still be vigilant in monitoring their investments and taking steps to prevent further losses.

More so, in Figures 2 and 4 respectively the presents of probability parameter can help to quantify the likelihood of different outcomes, such as positive or negative returns on investments. This can be useful for investors in assessing the risk of their investments and determining the optimal investment strategies. So by incorporating probability parameter into the model of wealth assessment, investors can better balance their portfolios by selecting a mix of investments with different levels of risk and return.

**4.1. Conclusion**

This paper considered approximate solution of Constant Elasticity of Variance equations. So four system of CEV equations were formulated in divers forms. The problems were solved analytical to obtain a closed form solutions of assessing different wealth of an independent corporate investors. From the stochastic approximation of graphical solutions portrays as follows: For solution 1: the relationship between the wealth and the returns is not perfectly linear. For solution 2: the wealth of investor is directly proportional to the returns of the investment. For solution 3: the wealth is growing at an increasing rate over time. Solution 4: the wealth of the investment is decreasing at steady rate over time. Finally, probability parameter quantified the likelihood of different outcomes such as positive or negative returns on investments.

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1.

2.

3.

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