**Stochastic Analysis of European Options and System of Linear Equations on the Analysis Of Share Prices for Capital Markets**

Abstract

This is about option pricing and some of its dynamics in financial markets via valuation using Black-Scholes (BS) model as it explores the changes of option (values) as a function of security and time. Consequently, the Black-Scholes model of European options on share price of Fidelity, Access and their future merged banks which gave closed form prices of Call and Put option prices with variations of maturity dates, average share prices as well as their respective standard deviations affecting real life changes for capital markets. From the share price analysis the growth rates of each bank were considered where Fidelity bank had the largest growth rates as this is informative to investors or management of the banks in terms of decision making. This paper offered here has reflective consequence for future studies of option prices.

**INTRODUCTION**

The capital market has been identified as an institution that contributes to the economic growth and development of emerging economies like Nigeria Capital market is made up of financial institutions which deal in long- term loans for investment. They therefore bring long-term lenders and borrowers together. Loans given are usually for more than two years. Institutions that operate in this market include; insurance companies, issuing Houses, Development Banks, Investment Banks, Investment Trusts, Building Societies or Mortgage Banks, Finance Corporations, Savings Banks and Stock Exchange. Financial instruments used in the capital market to finance long-term investments are stocks and shares, company bonds and government bonds The capital market can be divided into the primary and the secondary market. The primary market deals with the buying and selling of new securities. The secondary market is the market that deals with the buying and selling of already existing (secondhand) securities. It is dominated by the Stock Exchange. In the narrowest sense, the capital market involves the problems and prospect of equity investment. This involves the issue and market of shares, bonds and debentures using the services of brokers, dealers and underwriters. The capital market provides a means through which this is made possible. However, the paucity of long-term capital has posed the greatest. In capital investments monies are been invested in a business such that the return rates will be appropriately utilized to cover day-to-day trading activities expenses. An investor may need additional capital assets; in order to improve in its trading events. So capital investments are established on the basis of: to acquire additional capital assets for expansion which enables the business to increase in unit production, create new ideas on products or even add value to the business and to explore on technological advancements in order to increase efficiency and reduce costs and to replace worn-out assets. In absence of capital investments, trading business will definitely have a hard time getting off the ground; Azor et al.(2023).

However, since the beginning of the stock market, investors have been trying to gain an edge. The number-crunchers out there have long sought out a mathematical model, which could predict price fluctuation. If they could somehow figure out the secret formula, they could become rich beyond their wildest dreams. A team of economists, Fischer Black, Myron Scholes, and Robert .C. Merton, tried to do just that, and they came up with a powerful mathematical model for financial markets that contain derivative instruments. Lacking a creative name, this model became known as the Black-Scholes-Merton model. From this larger model, smaller models and equations were made based on the same assumptions. After years of developing the model, Robert Merton is attributed with first mentioning the ''Black-Scholes options pricing model'' in 1973. This theoretical model could help options market-makers properly price options on all types of financial instruments. Their work was so ground-breaking that 24 years later in 1997, Robert C. Merton and Myron Scholes won the Nobel Memorial Prize in Economic Studies for their work.Therefore ,Black-Scholes is a pricing model used to determine the fair price or theoretical value for a call or a put option based on six variables such as volatility, type of option, underlying stock price, time, strike price, and risk-free rate. The quantum of speculation is more in case of stock market derivatives, and hence proper pricing of options eliminates the opportunity for any arbitrage. There are two important models for option pricing – Binomial Model and Black-Scholes Model. The model is used to determine the price of a European call option, which simply means that the option can only be exercised on the expiration date.The Black-Scholes model is used to calculate the theoretical price of European put and call options, where an option is a contract for the right to buy and sell shares at a later date or within a certain period at a particular price. Therefore they assumed some features of the financial market, including:European-style options: The model supposes European-style options. Those can only be exercised on the expiration date. With American-style options it is possible to exercise the option at any time during the life of the option. Efficient markets: It is assumed that the stock’s behavior is like a random walk. Meaning, at any given moment in time, the price of the underlying stock can go up or down. The future stock price is independent from the past. The market movements cannot be predicted.

The Black-Scholes formula is a mathematical model to calculate the price of put and call options. Since put and call options are distinctly different, there are two formulas, which account for each option. Call options give the option holder the right to buy the underlying stock for an agreed upon price anytime between today and upon expiration. Traders that believe the underlying stock will go up over time buy these call options in the hopes of making money. On the flip-side, put options give the option holder the right to sell the underlying stock for an agreed upon price anytime between today and upon expiration. Traders that think a stock is going to go down can buy these put options in the hopes of making money if the stock goes does.

However, the mathematical modeling of stock option has called for great concern to all scholars; as it is expected that this time empirical approach will give answers to many problems. For instance; Fadugba and Nwozo (2013) had an empirical study on valuation of options. They applied CN FDM and concluded that CN is unconditionally stable, convergent and more accurate when pricing European option. Osu et al. (2009), studied stability analysis of stochastic model of stock market price. They applied the method of Black- Scholes analysis using crank Nicolson as a numerical scheme. In their research, stock prices were stable, and increase rate of the stock shares was also determined. Tangman et al. (2008) examined High Order Compact (HOC) Schemes for quasilinear parabolic PDEs to discretize the BS PDE for numerical pricing of European and American options. Along the line, Dremkova and Ehrdardt (2011) presented compact finite difference schemes to solve nonlinear BS Equations for American options with a nonlinear volatility function. In that research, it was discovered that the compact scheme cannot be applied effectively on American options, the study finally used a fixed domain transformation to obtain its results. During the same period, Song and Wang (2013) used symbolic calculator software to provide a numerical solution applying the implicit scheme of FDM. This research combined the time fractional BS equation with the conditions sufficient by the normal put options. Some years later, Uddin et al. (2015) presented results for European call and put options using FDM and Finite Element Method (FEM). In a recent study Zhang et al. (2006) studied the tempered fractional derivative to price a European double-knout barrier option. They analyzed characteristics of the three fractional BS models through resemblance with the classical BS model. Much earlier, Cortes et al. (2005) included a vital aspect of errors into the numerical solution. They introduced the Mellin transformation and proposed that the errors of composite Simpson’s rule or Euler’s method can be abstained while pricing the BS equation in real – world of financial derivatives. Company et al. (2006) also used Mellin transform and a delta-defining sequence of the involved established Dirac delta function to assign a numerical solution. Other empirical studies on numerical solution considered different aspects, for instance, Company et al. (2008) employed the semi-discretization technique to deal with the issues arising as a result of a nonlinear case of interest modeling option pricing with transaction costs. The nonlinear BS models has gotten the attention of researchers because of the rational assumptions including transaction costs, high volatility, illiquid markets and large investors preferences can as well be included. Abkudinova and Ehrhardt (2008) examined that the CN and the R3C scheme are the most accurate techniques to price the European call option. These study absorb different volatility problems in stock price, option price and its derivatives. Rao (2016) also applied numerical scheme to the generalized Black-Scholes models for European call option. The outcome showed that the second order accuracy in time and third order accuracy in space were obtained. Amadi et al. (2020) worked on Black-Scholes partial differential equation on stock market prices. They applied B.S analytical formula and Crank – Nicolson numerical method. It was observed that Black-Scholes and Crank-Nicolson are impossible to differentiate but in terms of precisions, Black-Scholes analytical value were found to be more adequate. Furthermore, Shim and Kim (2016) focused on the Black-Scholes terminal value problem and provided its solutions through the Laplace transform. This study claimed that the proposed method is simpler than the existing methods

Amadi et al. (2024). Amadi et al.(2024) studied the perception of European option which is geared towards valuation of financial assets, the application of share prices of Fidelity and Access banks which gave closed form prices of call options. The explicit price on the variations of maturity days is found accordingly.

In the same vein, Babasola et al.(2008) analyzed BS formula for the valuation of European options; Hermite polynomials were applied. They concluded that BS formula can easily be achieved devoid of the use of partial differential equation. In another study of BS, Shin et al.(2016) considered the BS terminal value problem and observed that their proposed method is better, simple than the previous methods. In the work of Rodrigo et al.(2006), time varying factor were incorporated in the explicit formula for different aspect of options with the aim of providing exact solution for dividend paying equity of option. In considering the stability of stock market price of stochastic model; Osu et al.(2010) applied Crank-Nicolson numerical scheme to BS model. The results showed stock prices being stable and its increasing rate of stock shares was obtained. However, Nwobi et al (2019) studied Black-Scholes model because of its biasness in mispricing options. They established a new technique of assessing pricing effects on the premise to reduce pricing bias.

However, reviews show that Black-Scholes have copious applications as seen in the literature. This present paper is aimed at pricing European options of Share prices Fidelity and Access banks, future merging and determining their growth-rates for capital market investments as this will add values in this dynamic area of mathematical finance.

The major issues of investors or owners of corporations is the inability to take an appropriate decisions when using Black-Scholes model of option pricing for applications. These issues may have arisen due to the formulation of problem or inability to add more models to the study or inability to understand the analytical solutions as to interpret accurately to the real world market. These discrepancies may lead to models not predicting closed form solutions for European options for proper decision making; which may not be to the best interest of option traders or investors or management of Fidelity and Access banks. In order to tackle the above problem we impose European options to value shares for Call and Put options and to determine their various growth-rates to realistically assess share prices for capital markets.

**MATERIAL AND METHODS**

In this Section, we present some rudimentary preliminaries touching the dynamics of study,hence we have as follows:

**3.1. Stochastic Processes**

It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

**Stochastic Differential Equations**

Here, consider a market where the underlying asset price  , on a complete probability space  is governed by the following stochastic differential equation:

 . (1.1)

**Theorem 1.1**: (Ito’s formula) Let  be a filtered probability space be an adaptive stochastic process on  possessing a quadratic variation with SDE defined as:



 and for 



Adopting theorem 1.1 comfortably solves the SDE in (3/1) with a given solution below:

 (1.2)

**3.2 The Black-Scholes Model**

The seminar paper on option pricing by Fisher Black and Myron Scholes in 1973 (Black and Scholes 1973) has had great impact on contemporary research in Mathematics of finance. This model is commonly used in financial modeling. The Black-Scholes model is made up of on seven assumptions:

* The asset price has characteristics of a Brownian motion with and  as constants.
* The transaction costs or taxes are not allowed.
* The entire securities are absolutely divisible.
* Dividend is not permitted during the period of the derivatives.
* Unacceptable of riskless arbitrage opportunities.
* The security trading is constant.
* The option is exercised at the time of expiry for both call and put options.

In mathematical finance, an arbitrage arguments show that any derivative  written on  must satisfy the partial differential equation of the form of option pricing; hence we have the following:

:

 . (1.3)

Where  represents interest rate , represents volatility of the underlying assets and  represents time of maturity.

With boundary conditions:

 . (1.4)

 . (1.5)

And final time condition given by :

 (1.6)

Equation (1.3) is the value of asset is worthless when the stock price is zero, Amadi et al.(2024). The details of the above option model can be expressly be found in the following books: Black-Scholes (1973), Hull (2012) etc.

To eliminate the price process in (1.3- 1.6) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

 (1.7)

where  is Price of a call option, is price of underlying asset, is the strike price,is the riskless rate ,is time to maturity, is variance of underlying asset, is standard deviation of the (generally referred to as volatility) underlying asset, and is the cumulative normal distribution.

Similarly Black-Scholes analytic formula for the prices of European Putl option is given as follows

 (1.8)

where is the price of a put option and the meaning of other parameters remain the same as in (1.7) Hull (2003).

However, the overall expected growth-rates for Fidelity, Access banks and their future merging is defined in a risk-neutral world as follows:



**RESULTS AND DISCUSSION**

This Section presents analyzed results whose methods are stated in above section. Hence we have the following parameter values: ****which were implemented using Matlab programming software:

**Table 1: The value of Share price of Fidelity Bank, PLC According to trading days for European Call Option**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Initial share price** | **The value of share prices when time** | **The value of share prices when time** | **Average**  **Value of share prices** | **STD** |
| 415 | 100 | 140 | 120 | 28.2843 |
| 62 | 0.016 | 0.445 | 0.2305 | 0.3033 |
| 138 | 1.653 | 8.00 | 4.8265 | 4.4880 |
| 61 | 0.014 | 0.417 | 0.2155 | 0.2850 |
| 121 | 0.852 | 5.232 | 3.0420 | 3.0971 |
| 81 | 0.088 | 1.268 | 0.6780 | 0.8344 |
| 139 | 1.713 | 8.188 | 4.9505 | 4.5785 |
| 80 | 0.081 | 1.209 | 0.6450 | 0.7976 |
| 384 | 81.09 | 119 | 100.0450 | 26.8064 |

**Table 2: The value of Share price of Fidelity Bank, PLC According to trading days for European Put Option**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Initial share price** | **The value of share prices when when time** | **The value of share prices when time when time** | **Average**  **Value of share prices** | **STD** |
| 415 | 407 | 413 | 410 | 4.2 |
| 62 | 52 | 53 | 53 | 0.7 |
| 138 | 130 | 133 | 132 | 2.1 |
| 61 | 51 | 52 | 52 | 0.7 |
| 121 | 113 | 116 | 115 | 2.1 |
| 81 | 72 | 73 | 73 | 0.7 |
| 139 | 131 | 134 | 133 | 2.1 |
| 80 | 71 | 72 | 72 | 0.7 |
| 384 | 376 | 382 | 379 | 4.2 |

**Table 3: The value of Share price of Access Bank, PLC According to trading days for European Call Option**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Initial share price** | **The value of share prices when when time** | **The value of share prices when time when time** | **Average**  **Value of share prices** | **STD** |
| 410 | 97 | 137 | 117 | 28.28 |
| 80 | 0.08 | 1.21 | 0.64 | 0.80 |
| 126 | 1.05 | 5.98 | 3.52 | 3.49 |
| 79 | 0.075 | 1.15 | 0.61 | 0.76 |
| 98 | 0.271 | 2.54 | 1.41 | 1.60 |
| 92 | 0.188 | 2.028 | 1.11 | 3.30 |
| 127 | 1.093 | 6.13 | 3.65 | 3.56 |
| 91 | 0.176 | 1.95 | 1.06 | 1.25 |
| 378 | 77.57 | 114.1 | 96 | 25.83 |

**Table.4: The value of Share price of Access Bank, PLC According to trading days for European Put Option**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Initial share price** | **The value of share prices when when time** | **The value of share prices when time when time** | **Average**  **Value of share prices** | **STD** |
| 410 | 402 | 408 | 405 | 4.2 |
| 80 | 71 | 72 | 72 | 0.7 |
| 126 | 118 | 121 | 110 | 2.1 |
| 79 | 70 | 71 | 71 | 0.7 |
| 98 | 90 | 92 | 91 | 1.4 |
| 92 | 84 | 85 | 85 | 0.7 |
| 127 | 119 | 122 | 121 | 2.1 |
| 91 | 83 | 84 | 84 | 0.7 |
| 378 | 370 | 376 | 373 | 4.2 |

Tables 1-4 shows the interpretations of Call and Put options with different maturity dates in terms of investment plans. Different maturity dates can give investor or the banks more flexibility to adjust positions based on changing market conditions. For example, if the banks are bullish on a stock in the long-term but bearish in the short-terms they could buy a Call option with a long-term maturity date while simultaneously selling a Call option with a shorter term. Also options with different maturity dates can be used to hedge against potential losses.

In columns 4 and 5 of Tables 1.6 are the value of averages and standard deviations respectively. The average share price can be used to determine whether a stock is overvalued or undervalued, which can help investors make decisions about when to buy or sell. Looking at the average share prices of Fidelity and Access banks over time can help investors identify patterns and trends that may be useful for making investment decisions. The average share price can be comparing different stocks or sectors , which can be helpful for diversifying a portfolios.

Share price standard deviation is measure of volatility or the extent to which a data set varies from its mean value. In the context of Call and Put options with different maturity dates, standard deviation tells how much the price of the options is likely to fluctuate over time. A higher standard deviation indicates that the options are more volatile and may be riskier to hold. A low standard deviation indicates that the options are more stable and may be less risky. Standard deviation can be important consideration for Fidelity and Access banks who are trying to manage their risk to make predictions about future price movements; see Tables 4-6 columns 5

**Table 5: The value of Share price of Fidelity-Access Merged According to trading days for European Call Option**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Initial share price** | **The value of share prices when time when time** | **The value of share prices when time when time** | **Average**  **Value of share prices** | **STD** |
| 825 | 449 | 491 | 470 | 29.70 |
| 142 | 1.89 | 8.76 | 5.33 | 4.86 |
| 264 | 24.93 | 50 | 37.47 | 17.73 |
| 140 | 1.77 | 8 | 4.89 | 4.41 |
| 219 | 12.51 | 31 | 21.76 | 13.07 |
| 173 | 4.72 | 16 | 10.36 | 7.98 |
| 266 | 25.60 | 51 | 38.30 | 17.96 |
| 171 | 4.48 | 15 | 9.74 | 7.44 |
| 762 | 389 | 432 | 411 | 30.41 |

**Table 6: The value of Share price of Fidelity-Access Merged According to trading days for European Put Option**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Initial share price** | **The value of share prices when time when time** | **The value of share prices when time when time** | **Average**  **Value of share prices** | **STD** |
| 825 | 817 | 823 | 820 | 4.2 |
| 142 | 134 | 137 | 136 | 2.1 |
| 264 | 256 | 261 | 259 | 3.5 |
| 140 | 132 | 135 | 134 | 2.1 |
| 219 | 211 | 216 | 214 | 3.5 |
| 173 | 165 | 169 | 167 | 2.8 |
| 266 | 258 | 263 | 261 | 3.5 |
| 171 | 163 | 167 | 165 | 2.8 |
| 762 | 754 | 760 | 757 | 4.2 |

Tables 5 and 6 shows the value of Fidelity and Access banks share prices as they merge in future. It can be seen from the variations of maturity days that increase in the maturity days increases the value of call option prices. Careful looking at the call option prices as they merge; one will understand that it is more profitable for the two banks to merge because the value of their assets will increase tremendously as seen above. This remark is encouraging in every investments because it is profit maximizing which will guide the management of banks , the ways of taking decisions based on the levels of their investments.

**Table 7: The share price growth-rates of Fidelity,Access banks and their future merging**

|  |  |  |
| --- | --- | --- |
| **Fidelity Bank ,PLC** | **Access Bank,PLC** | **Future Merged** |
| 190 | 163 | 176 |
| 80 | 45 | 62 |
| 81 | 46 | 63 |
| 68 | 22 | 45 |
| 40 | 6.0 | 24 |
| 53 | 32 | 43 |
| 55 | 33 | 44 |
| 157 | 142 | 149 |

Table 7 represents some levels of growth-rates from Fidelity, Access and their future merge banks. As can be seen that a higher growth rates means that a share price has been increasing at a faster rate over a certain period of time. Shares in this table with higher growth rates may be attractive to investors because they have the potential to generate higher returns. Whereas smaller growth rate means that a share price has been increasing at a a slower rate over a certain period of time. Shares with smaller growth rates may be less attractive to investors because they have less potential for generating high returns. In all Fidelity bank has largest growth rates and profit indexed in terms of returns.

**CONCLUSION**

This paper studied the framework of Black-Scholes model of European options on share price of Fidelity, Access and their future merged banks which gave closed form prices of Call and Put option prices with variations of maturity dates, average share prices as well as their respective standard deviations affecting real life changes for capital markets. From the share price analysis the growth rates of each bank were considered where Fidelity bank had the largest growth rates as this is informative in terms of decision making.

However, the current study is on European options case. Another study could be considered in the case of multi-options in one portfolio of investments.

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