**Solution of Certain System of Ordinary Differential Equations using “Saxena & Gupta Transform”**

**Abstract:** Integral transforms play an important role in solving system of ordinary differential equations and integral equations. In the present paper we discuss some applications of new transform “Saxena & Gupta” transform is an interesting method to solve certain type of system of ODEs. This transform operates by converting a given system of ordinary differential equations into an algebraic equations. Upon solving this algebraic equations, the inverse transform is applied to yield the sought-after solution.

**Keywords** : Saxena & Gupta transform, inverse Saxena & Gupta transform, system of differential equation, Boundary value problems.

1. **Introduction**

Fractional calculus is the branch of mathematics which deal with the investigation and applications of integrals and derivatives arbitrary order. Due to the growing range of applications, there has been significant interest in developing transforms for the solution of fractional differential equations.

Integral transforms are the most useful techniques of the mathematics which are used to find the solutions of differential equations, partial differential equations, integro-differential equations, partial integro- differential equations, delay differential equations and population growth.

In this paper we apply a new integral transform, called Saxena & Gupta transform, for solving a system of ordinary differential equations. Integral transformations essential for solving complex problems in engineering, natural sciences, computers, optical sciences, and modern mathematics to a simple system of algebraic equations that can be solved easily.

**Definition :**

 **Let**  be a function of . The new transform of a function is defined as follows, see in [ 7 ]

 ………………. (1.1)

The above integral convergent.

**Saxena & Gupta transform of derivatives :-**

**Table 1--** **Saxena & Gupta transform of some elementary function**

|  |  |  |  |
| --- | --- | --- | --- |
| S.NO. |  | Function | New transform |
| 1 |  | 1 |  |
| 2 |  |  | 1 |
| 3 |  |  | 2ʋ |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |

1. **Application of Saxena and Gupta Transform of Certain system of Ordinary Differential Equations.**

 As specified in the introduction of this paper, the Saxena and Gupta transform can be used as an effective tool. For analysing the basic properties of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of the Saxena and Gupta transform in solving certain initial value problems described by system of ordinary differential equations[1],[2],[3]

**Theorem (2.1)-** Consider the system of differential equations

.

 ......……………. (2.1.1)

 ….……....….... (2.1.2)

 with initial condition

 **Solution :** To obtain the solution of system of ordinary differential equations first weApplying the Saxena & Gupta transform of both side of eq. (2.1.1) and (2.1.2)

since

 + ……………..(2.1.3)

 - ………………(2.1.4)

Solving this equations for

 …………… (2.1.5)

 …………… (2.1.6)

Applying inverse Saxena & Gupta transforms

since

Thus required solution of given differential equations are

 ………….. (2.1.7)

 …………… (2.1.8)

**Theorem -(2.2)-** Find the solution of the system of the differential equations

 …………… (2.2.1)

 ……........ (2.2.2)

with initial conditions , where are arbitrary constants

**Solution :**To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.2.1) and (2.2.2), we get

since

 + …………(2.2.3)

Solving this equations for

 ………………. (2.2.5)

 ………………… (2.2.6)

Applying the inverse Saxena & Gupta transform both sides of the equation (2.2.5) and (2.2.6)

since

 thus required solution of given differential equations are

 ……………. (2.2.7)

 …………….. (2.2.8)

**Theorem -(2.3)-** Find the solution of the system of ordinary differential equations

 …………… (2.3.1)

 ……...........(2.3.2)

with initial conditions

**Solution** : To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.3.1) and (2.3.2), we get

since

 …………….. (2.3.3)

 +2 ………………(2.3.4)

Solving this equations for

 ………………. (2.3.5)

 ……………….. (2.3.6) Applying the inverse Saxena & Gupta transform both sides of the equation (2.3.5) and (2.3.6)

since

 thus required solution of given differential equations are

 …… (2.3.7)

 …….. (2.3.8)

**Theorem -(2.4)-** Find the solution of the system of ordinary differential equations

 …………… (2.4.1)

 ……...........(2.4.2)

with initial conditions

**Solution :**To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.4.1) and (2.4.2)

since

 …….. (2.4.3)

 + ………. (2.4.4)

Solving this equations for

 ………. (2.4.5)

 …….. (2.4.6)

Applying the inverse Saxena & Gupta transform both side of the equation (2.4.5) and (2.4.6)

since

 thus required solution of given differential equations are

 …………… (2.4.7)

 ……………. (2.4.8)

**Theorem -(2.5)-** Find the solution of the system of ordinal differential equations

 …………… (2.5.1)

 ..……...........(2.5.2)

with initial conditions

**Solution:**- **:** To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both side of eq. (2.5.1) and (2.5.2)

since

 …….. (2.5.3)

 =2 …. (2.5.4)

Solving this equations for then applying inverse transforms we get the solution of given differential equations are

 ……………… (2.5.5)

 ………………. (2.5.6)

**Theorem -(2.6)-** Find the solution of the system of the equations

 …………… (2.6.1)

 ……...........(2.6.2)

With initial conditions

**Solution:**- : To obtain the solution of system of ordinary differential equations first weapplying the Saxena & Gupta transform of both sides of eq. (2.6.1) and (2.6.2)

 )

since

 ….. (2.6.3)

 … (2.6.4)

Solving this equations for

 ………. (2.6.5)

 ……….. (2.6.6)

Applying the inverse Saxena & Gupta transform both side of the equation (2.6.5) and (2.6.6)

since

 thus required solution of given differential equations are

 ……………. (2.6.7)

 …………. (2.6.8)

**Conclusion :** This innovative technique demonstrates greater effectiveness and ease of use in handling ordinary differential equations compared to conventional methods. Also this method is very efficient, simple and engineering applications , with the potential to extend its utility to a wide array of problems across various domains. The main goal of this research is to solve certain system of ordinary differential equations.

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