**SOLUTION AND ANALYSIS OF A DYNAMIC CASE OF A FINITE DEFORMATION OF A HOLLOW CYLINDRICAL TUBE OF A NATURAL VULCANIZED RUBBER UNDER INTERNAL PRESSURE**

**Abstract**

In this work, the dynamic case of the deformation of internally pressurized hollow cylinder made of natural vulcanized rubber material is considered. The analysis of the deformation led to non-linear second order partial differential equation for the determination of stresses and displacement. Knowing that most of wave equations are weakly non-linear and as such their solution are time independent. We employ monge method which reduces the equation into linear second order ordinary differential equation where we sought for D operator Method of solution and appropriate boundary conditions were applied for the determination of closed form solution of the displacement and stresses at various parts of the cylindrical tubes..

Keywords: Vulcanized rubber, displacement, undeformed radius, stresses and Hollow cylinder.

1. **Introduction**

Cylindrical materials such as tyres, hydraulic hoses, wires, shocks, tubes Seals, vibration absorbers, their mechanical properties play important role in their usage. Most Cylindrical tubes are subjected to internal pressures and as a result, they undergo finite deformations. Therefore, a good understanding and knowledge of deformations mechanism of many structures and materials under different loading conditions are to be known because of great important to materials testing and products development. The analysis of the dynamic case of finite deformation of internally pressurized hollow cylinder of a vulcanized rubber material is considered. The aim of the work is to determine the displacements and stresses caused by the internal pressure p(t) which is time dependent at any cross section of the hollow cylinder. Erumaka (1) worked on internally pressurized vulcanized rubber where he established a condition for a non trivial solution of the displacement. Nwagwu (2) also worked on internally pressurized vulcanized rubber and obtained a closed form solution for the displacement in his work it was noted that as the radius of the hollow cylinder made of vulcanized rubber increases the displacement of the material increases and that the maximum displacement may not be possible until collapse of the material. Ejike and Erumaka (3) worked on deformation of a rotating circular cylinder made of Blatz-ko material and considered two different cases hollow and solid cylinder and were able to obtain an approximate solution for the displacement. Erumaka et. al (4) investigated on axial shear wave in an incompressible solid and obtained a closed form solutions for the displacement and stresses across various parts of the cylindrical material. Huang (5) worked on finite displacement of a hollow sphere under internal and external pressures. Erumaka et. al (6) did a work on the title Azimuthal Shear Wave in an Incompressible Hollow Circular Cylinder and achieved a closed for solution for the displacement and stresses. Their results support that incompressible material does not depend on time.Aani and Rahimi (7) investigated on the displacement and stresses of axisymmetric radial deformation of the shell. They employed the use of Neo-Hookean strain energy function to obtain the behaviour of the material. The analysis and results presented shown the effect of the stress generated for an internally pressurized thick walled cylinders containing an internal radial hole using finite element method. Darijani and Bahremen (8) employed polynomial hyperelastic models to obtain a closed form solution for analyses of rubbery solid circular cylinder.

Elkholy et. al (9) study on the effect of Finite Element Analysis of Stresses Caused by External Holes in Hydraulic Cylinders. Erumaka et. al (10) worked on the title Combined Axial and Azimuthal Shear Wave in an Incompressible Hollow Circular Cylinder were they obtained the combined displacements through the principle of superposition**.** Gao(11) analysed Elasto-plastic analysis of an internally pressurized Elasto-plastic analysis of an internally pressurized thick-walled cylinder using a strain gradient plasticity theory. The numerical data presented demonstrated that the classical plasticity-based solution and the gradient plasticity-based solution predict almost identical results. Fracture mechanics analysis of cylindrical pressure vessels was also carried out.. Anani and Gholamhosein(12) worked on spherical material, Stress analysis of thick pressure vessel composed of incompressible hyperelastic materials where Neo Hookean strain energy function was used to determine the stress and displacement of spherical shell that is axisymmetric radially deformed under internal and external pressure. Nabham et. al., (13) study the effect of the stress generated for an internally pressurized thick walled cylinders containing an internal radial hole using finite element method. Their results shown that hoop stress increases due to increase of the hole parameter, depth and diameter. Furthermore, the characterizations of notch may be used to determine the maximum stress limit. Aani and Rahimi (14) investigated the stability of internally pressurized thick-walled spherical and cylindrical shells made of functionally graded incompressible. Chung et al (15) determined the deformation of internally pressurized hollow cylinder and spheres for Blatz-ko type of compressible elastic material. The results presented shown that, when the ratio of the outer undeformed radius to the inner undeformed is higher than the critical value, the shear bifurcation occurs before the maximum pressure is reached, they also shown that the reverse occurs when the ratio is lower than the critical value. In this present paper knowing that most of wave equations are weakly non-linear and as such their solution are time independent. We employ Monge method which reduce the equation into linear second order ordinary differential equation and we sought for a closed solution using D-operator method of solution of second order ordinary differential equation for the determination of stresses and displacement across a hollow cylindrical pipe made of vulcanized rubber material.

**2. Governing equations**

Let consider an open region denote the cross section of a right circular tube with inner radius a and outer radius b in its initial configuration . The cylindrical tube is subjected to a time dependent internal pressure of magnitude . The resulting deformation is a one to one axisymmetric deformation which maps the point with cylindrical polar coordinate in the initial configuration to the point in the current region such that

 (1)

where (R,t) is radial displacement, R and r are radii of the cylinder in the initial and current configurations respectively. where is to be obtained

a

b

 (1

The deformation gradient tensor for the given equation (1) is

 (2)

The Left Cauchy-Green deformation gradient tensor associated with the given (1) is

 (3)

 (4) is the transpose of

Then equation (4) becomes

 (5)

=

Where the three principal strain invariants , and have their usual formulas as

 = (6.1)

 (6.2)

 (6.3)

 is the strain energy function and =

Here we consider compressible isotropic elastic vulcanized rubber material characterized by the elastic potential

 (7)

where is shear modulus, I1 and I3 are the first and third principal invariants respectively.

 (8)

Evaluating in (8) using (6), we obtain the following:

 (9)

 (10)

 (11)

**3 Stress Tensor T:** Considering the stress tensor T for compressible material as

 (12)

where

 (13.1)

 (13.2)

 (13.3)

Since

 (14a)

Evaluating (14a) using (5) and (13), we obtain

)

 (14b)

The stress tensor T in cylindrical polar form is given as:

 (15)

By comparing (14b) with (15), we obtain the components of stress tensor as

 (16a)

 (16b)

 (16c)

 (16d)

Where

**4 Equations of motion:** The equations of motion in cylindrical polar co-ordinates is given as:

 (17a)

 (17b)

 (17c)

Here, , and bz are the components of the body forces,, and az are the components of the acceleration and is the material mass density. Based on the deformation equations in (3.1), the motions are on radial direction, that is, ar ≠ 0 and

The non-zero components of the equations of motion is given as

 (18a)

Substituting (16) in (18), we have

+

+

where

 (18b)

1. **MONGE METHOD OF SOLUTION**

The standard form of the Monge equation is given as

 (19)

where .

where

comparing (18) and (19), we have

 ,

Monge Subsidary equations

 (21)

Substituting the values of in (20) and (21) we have (22) and (23)

 dRdt=0 (22)

Equation becomes

 (24)

Substituting (24) into (22) we have (25a) and (25a)

 dRdt=0 (25a) dRdt=0 (25b)

 (25a)+(25b) becomes

 )dRdt=0 (26)

Equation (26) reduces to

 where

Let

where

Let

 (28)

Therefore

Using (14) and (15), (13) becomes

Let

Then (7) becomes

 (30)

The auxillary equation of equation (30) is given as

 (31)

1. **Boundary conditions**

Using boundary conditions in terms of the displacement of the form

It is important to note that the above boundary conditions must satisfy

Then we obtain the values of A and B in equation (31) as

 and

substituting the value of and B in equation (31) we have

 (32)

For steady state then equation (32) becomes

Considering the case of constant velocity where becomes

 (33)

1. **Result and Analysis**

Table 1: Table of values for displacement, undeformed radius and time generated from (33).

|  |  |  |
| --- | --- | --- |
| Time | Undeformed radius, R | Displacement, r(R, t) |
| 0 | 10 | 0.0000 |
| 1 | 12 | 0.8098 |
| 2 | 14 | 1.3191 |
| 3 | 16 | 1.6072 |
| 4 | 18 | 1.7223 |
| 5 | 20 | 1.6953 |
| 6 | 22 | 1.5474 |
| 7 | 24 | 1.2938 |
| 8 | 26 | 0.9461 |
| 9 | 28 | 0.5126 |
| 10 | 30 | 0.0001 |

Figure 1: A graph of displacement against undeformed radius for a hollow cylindrical material.

The graph shows the mathematical description of a travelling wave

Figure 2: A graph of displacement against time

Mathematical description of travelling waves moving lines in time.

Table 2: Table of values for displacement, undeformed radius and time for the case of constant velocity. Note that in table 1 stress is undefined at t=0 which can be due to pre stressed in the material even though the initial displacement is zero and vice versa

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| time, t | undeformed radius, R (mm) | Displacementr(R, t) |  |  |
| **1** | **10** | **0.9999** | **-0.1087** | **0.4993** |
| **2** | **12** | **1.6192** | **-0.1674** | **0.4993** |
| **3** | **14** | **1.9779** | **-0.2031** | **0.4993** |
| **4** | **16** | **2.1420** | **-0.2270** | **0.4992** |
| **5** | **18** | **2.1515** | **-0.2443** | **0.4991** |
| **6** | **20** | **2.0325** | **-0.2578** | **0.4990** |
| **7** | **22** | **1.8030** | **-0.2977** | **0.4987** |
| **8** | **24** | **1.4759** | **-0.2785** | **0.4985** |
| **9** | **26** | **1.0609** | **-0.2871** | **0.4979** |
| **10** | **28** | **0.5655** | **-0.2950** | **0.4961** |
| **11** | **30** | **0.0044** | **-0.3025** | **0.0000** |

Figure 3: A graph of stress against displacement

The graph shows that vulcanized rubber is a highly compressible material. It can undergo large deformation without reaching to its breaking point

Figure 4: A graph of stress against undeformed radius

The graph shows that stress will become zero irrespective of the input variable

**Figure 5: A graph of stress against time**

The graph shows that as time t, progresses the stress must tend tto zero

1. **Conclusion**

In this work we were able to establish a closed solution for the displacement and stresses for the dynamic case of internally pressurized hollow cylindrical pipe made of vulcanized rubber material. Table 1 showed that at initial time the stress can become undefined due to pre stressed in the material even when the displacement is zero. The graph of figure 3 shows that vulcanized rubber is a highly compressible material which can undergo large deformation without reaching to its breaking point. The graph of figure 5 shows that as time t, progresses the stress must become zero. Figure 2 describe the Mathematical description of travelling waves moving lines in time. A graph of undeformed radius against displacement is plotted as shown in figure 1. Equations (33) and (16) give the displacement and components of the stress and table 2 represents values of displacement and stresses at a certain cross section of the hollow cylindrical pipe made of vulcanized rubber material. It was observed that as the radius of the hollow cylinder made of vulcanized rubber increases, the deformation gradient decreases. The result shown that the radius of the hollow cylinder made of vulcanized rubber increases, the displacement of the material increases.

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