**Stochastic Model on Share Price Movements in Finite State with Asymptotic Null Controllability Properties**

**Abstract**

In this paper, a stochastic model of Markov chain and asymptotic null controllability properties for share prices were considered for decision making. The three-step transition probability and controllability matrices were developed and analyzed for independent banks such as Access and Fidelity, respectively. The précised measures which govern future share price changes for prediction were obtained. Furthermore, controllability analyses were conducted to demonstrate the non-singularity of the controllability matrices regarding the share prices of the two banks. This indicates to investors that the financial market is poised for stability in the long term, which constitutes the main focus of this paper. Several examples were provided to illustrate the dependability and efficiency of the model.

**Keywords: Share price, Markov Chain, Access and Fidelity, Stochastic analysis and Controllability matrix.**

**1 Introduction**

It is clear that the fluctuation of stock prices, whether observed over short or extended trading periods, is considered a stochastic phenomenon that can be effectively modeled for empirical investigations. With its unique traits characterized by fluctuating share prices, the financial market in Nigeria places significant emphasis on the Nigerian Stock Exchange (NSE) as a crucial avenue for capital raising and facilitating transactions between companies and investors. Empirical research centered on the NSE indicates that a thorough understanding of stock price dynamics can offer valuable insights (Agwuegbo et al., 2010). Therefore, the performance of the stock market and its related operations carry substantial importance as a viable and impactful investment sector within the broader financial landscape. Numerous scholars have extensively contributed to the discourse on stock market price formations, as evidenced by a range of works (Amadi et al. (2022); Davies et al., (2019); Ofomata et al., (2017); Ugbebor et al., (2001) & Adeosun et al. (2015)) among others. The evolution of prices in the realm of risky assets is often envisioned as the trajectory of a Markov process navigating within a specified state space determined by transition probabilities. In a stochastic context, the Markov chain methodology employs a set of transition matrices that illustrate the random progression from one state to another. This underscores the noteworthy feature of Markov chains, characterized by their lack of memory, where the future state is solely dependent on the current state and not influenced by the historical sequence of events over time. Markov chains have emerged as a prominently developed theory within stochastic processes, finding wide-ranging applications in the continually expanding domains of science and technology.

Numerous scholars have extensively explored the utilization of Markov chains in modeling stock prices, yielding diverse findings. Mettle et al., (2014) specifically delved into the stochastic analysis of share prices, delineating precise conditions to determine the expected mean return time for stocks. This enhances investment decision-making by focusing on the highest transition probabilities. Similarly, Agwuegbo, et al., (2010) investigated the dynamics of stock market prices, analyzing their fluctuations and their broader impact on a nation's financial well-being and economic vitality. Their conclusions underscored the stochastic nature of stock prices, affirming that investors cannot influence the fairness or unfairness of stock prices based on expectations.

In their research endeavors, Bairagi and Kakaty (2015) delved into examining stock market behavior using Markov chains, revealing a notable finding: the steady-state probabilities of share prices remained constant regardless of the current price of a bank's shares. Similarly, Zhang and Zhang (2009) introduced a tailored Markov chain model for predicting stock market trends, demonstrating its efficacy in analyzing and forecasting both market indices and closing stock prices. Turning to the long-term outlook of security prices in Nigeria, Eseoghene (2011) gathered data from various banks in the Nigerian banking sector, suggesting potential stability in long-term price levels regardless of prevailing circumstances. Additionally, Christian and Timothy (2014) investigated the enduring patterns of closing prices for shares from eight Nigerian banks, utilizing a Markov chain model. They observed a positive outlook for Nigerian bank stocks despite current market conditions. The conclusion emphasized the potential utility of the study's results for investors.

However, controllability stands as a qualitative characteristic within dynamical systems, holding particular significance in control theory. This concept has been instrumental in deterministic system theory, where the controllability of equations is extensively employed for analysis and control system design. In the realm of control systems, controllability asserts that every state of a process can be influenced or manipulated by specific control signals within a designated timeframe. This implies the capability to guide the system from any past state to any future state using permissible controls within a finite duration. Systems that possess complete controllability demonstrate this ability. However, if complete controllability is not attained, different types of controllability, such as approximate, null, local null, and local approximate null controllability, can be considered. The controllability property holds significance in addressing a range of control problems within a control system. (Osu et al. 2019)

In their investigations, Davies et al. (2023) focused on analyzing the stability and controllability of stock market prices, utilizing stochastic vector differential equations with control measures. Their findings indicated the stability of stock prices, with results showing asymptotic null controllability. Meanwhile, Bullo et al. (2019) delved into controllable kinematic reduction for mechanical systems, concentrating on a specific subset of systems with constraints modeled as connection control systems. They highlighted reduction techniques and controllability conditions expressed through a particular vector-valued quadratic form. Additionally, Davies (2005) explored the relative controllability of nonlinear systems with delays in both state and control variables. Their study established sufficient conditions for Euclidean controllability in perturbed nonlinear systems with time-varying multiple delays in control, utilizing Dabo's fixed point theorem to analyze the perturbed function.

In another research endeavour, Davies (2006) focused on investigating the Euclidean null controllability of linear systems incorporating delays in both state and control variables. The findings established conditions conducive to achieving Euclidean controllability in such systems. Similarly, Manikonda and Krishnaprasad, (2002) examined the controllability of under-actuated mechanical systems with symmetry, leveraging controlled nonlinear dynamics' invariance to group action. Through geometric mechanics, they derived reduced dynamics for the system, shedding light on its controllability. Additionally, Matyukhin (2004) explored the controllability of non-holonomic mechanical systems with constrained controls, emphasizing physically meaningful controllability conditions. In a related study, Matyukhin (2005) delved into the controllability of mechanical systems, considering the dynamics of control drives, emphasizing the importance of rapid changes in control output, particularly in control forces.

In the research conducted under Matyukhin and Pyatnitskii (2004), the focus was on assessing the controllability of mechanical systems characterized by controls linked with their derivatives. This investigation broadened controllability principles to encompass nonlinear dynamics and mechanical systems alike. The key discovery indicated that for a manipulative robot to qualify as controllable, the influence of control forces must surpass that of other generalized forces, such as gravitational forces or environmental resistance.

This study delves into the stochastic analysis of Markov chains applied to the closing share price data of Access and Fidelity from 2016 to 2022. The share prices were transformed into a 3-step transition probability matrix solution, spanning the specified time frame. The insights gained into future share price changes serve as a valuable decision-making tool for the day-to-day management of the banks. Additionally, controllability analyses were conducted to demonstrate the non-singularity of the controllability matrices for the share prices of both banks. This focal point of the paper builds upon and extends the findings of previous works by Osu et al. (2019) and Davies et al (2023), respectively.

The aim of this paper is first, to present the share prices of Access and Fidelity in finite state and analyzing the asymptotic null controllability properties as it affects the share prices of two banks under-study.

The structure of this paper unfolds in the following manner: Section 2 provides an overview of materials and methods, while Section 3 elucidates the results and engages in discussions. The conclusion of the paper is encapsulated in Section 4.

**2. Material and Methods**

To comprehend the content of this Markov chain paper, we commence by elucidating the concept of a stochastic process. A stochastic process is essentially a statistical occurrence that unfolds over time based on probabilistic principles. In mathematical terms, it can be defined as an assemblage of random variables arranged chronologically at distinct time points, which may be either continuous or discrete. Since a stochastic process involves a set of random variables, its specifications align with those of random vectors.

**Definition 1:**  If the Markov property is met, a stochastic process X is deemed a Markov chain.

  (1)

For all ** .**

Understanding that the Markov property defined in equation (1) is equivalent to any of the following conditions for each $j\in S$ is adequate.

  (2)

 

If $X\_{n}=i$ is assumed, indicating that the chain is in the ith state at the nth step, it can also be expressed as the chain having the value i or being in state i. The concept underlying the chain is explained through its transition probabilities:

  (3)

They are dependent on ****

**Definition 2:** The chain is said to be homogeneous if the following are stated below

  (4)

For all ****

The transition matrix  is  matrix of transition probabilities.

  (5)

Therefore, in a homogeneous Markov chain, the transition probabilities consistently remain fixed and unchanging over time.

**Theorem 1:** Suppose  is a stochastic matrix which implies the following:

i) has non-negative entries or  (ii)

which is stationarity or point of convergence.

Proof:( i) each associated entry in  is a transition probability and being probability .

(ii) 

Which is stationarity.



**Theorem 2 :( Chapman-Kolmogorov Equations).**

**** Since and so on the  power of  .

Proof: 

Using the following in probability rule:

 and setting 

Using Markov property yields



To obtain an estimates of the transition probability as follows

 

 

where is the number of states.

  (6)

**2.1 Developing Markov Chain Model for Stochastic Analysis of Access, Fidelity Share Prices**

To ensure the precise efficacy of the Markov chain model in forecasting future events, it is essential to construct a predictive framework for the movement of share prices. The initial share prices must be categorized into three distinct finite states as outlined below:

**R:** signifies the probability of share price dropping in near future

**I** : denotes the probability of share price growing in near future

**NO-change** : symbolizes the probability of share price not altering in near future

Nevertheless, the transition matrix's probability provides a comprehensive representation of the Markov chain, where each element in the matrix conveys meaningful information. To establish the three states of the Markov process, the requisite table is outlined below.

**Table 1: Transition Probability Matrix**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **State** | **1** | **2** | **3** | **Total of Row** |
| **1** |  |  |  |  |
| **2** |  |  |  |  |
| **3** |  |  |  |  |

However, for  is an estimate of the transition matrix.

  (7)

  (8)

  (9)

Setting 

**2.2 Controllability Analysis of Share Prices**

This Section deals with the controllability analysis of each transition probability (7-9) when control measures are introduced such that:

A stochastic process  is said to be a Markov chain if the Markov property is satisfied :

  (10)

for all ** .**

where  represents time-dependent control measures

Introducing time-dependent control in (7-9) gives the following



  (11)

  (12)

where  represents time-dependent controls of each share price.

To effectively study the impact of control measures on the share price movements, we therefore define the following  to have:

  (13)

Define  and the controllability matrix

  (14)

where  denotes the transpose of the matrices of each share prices, following [15]. We assume that the following limits exist:



**Theorem 5:** equation (10) is null controllable if and only if  is non-singular. The proof of controllability is seen in Davies et al. (2023).

**3. Results and Discussion**

To illustrate the share price of two banks' movement in finite states using Markov chain model extracted from Osu et al. (2019) were used for the study

**Probability Transition Matrix For Access Bank Share Price**

 

**Access bank (2016-2022)**: Signals suggest a 67% chance of a decline in prices, a 13% likelihood of price upswing, and a 20% probability of price stability in the near future. Likewise, in similar situations, there exists a 29% probability of a price drop, a 36% chance of an increase, and a 34% likelihood of no alteration. Wrapping up the scenarios, there is a 21% probability of a price fall, a 15% chance of an increase, and a 63% likelihood of status quo in prices. This data provides valuable insights into potential future movements in the price of the subject, aiding strategic decision-making. The above valuations provide an eye-opener to the management of Access Bank, PLC that will enhance investment decisions.

**Probability Transition Matrix For Fidelity Bank Share Price**

 

**Fidelity (2016-2022)**: Expressed as probabilities, there is a 67% likelihood of a price reduction, a 10% chance of price increase, and a 22% chance of no change in the near future for one scenario. In a parallel context, another scenario indicates a 23% probability of price reduction, a 46% chance of price increase, and a 31% likelihood of no change. Finally, a third scenario presents a 23% chance of price reduction, a 13% probability of price increase, and a 64% chance of no change. The stochastic analysis of Fidelity's share prices offers insights into future price movements, contributing to informed investment decision-making over the long term.

**3.1 Illustrations of Asymptotic Null Controllability Results for Share prices of Access and Fidelity Banks**

Considering (11) and (12) we have the following:

 



 Which was solved following the method of [15] given as:

 

We show the nonsigularity of the controllability matrix Access bank using theorem 5 seen in Davies et al. (2023) as follows.

 let  and  respectively.

 

 

Similarly for Fidelity share prices , we also have as follows

 

We also show the nonsigularity of the controllability matrix Fidelity bank using theorem 5 seen in Davies et al. (2023) as follows.

 

 

Ultimately, the factors influencing the controllability matrices are non-singular, implying that the controllability of share prices for Access and Fidelity tends towards zero asymptotically. As the share prices of both banks approach asymptotic null controllability, it signifies long-term stability in the financial market, offering various advantages. Primarily, it instills investors with assurance regarding the future value of their investments. Furthermore, this phenomenon aids in mitigating market volatility as investors anticipate eventual price stabilization. Additionally, it enhances the market's resilience in rebounding from shocks and disruptions. Consequently, asymptotic null controllability emerges as a favorable attribute for financial markets.

**4. Conclusion**

In this paper, results for stochastic model and asymptotic null controllability for share prices were effectively obtained. At first, by developing 3-steps transition probability matrix with control for Access and Fidelity (2016-2022). The analysis of transition probability matrices indicates that Access Bank, PLC exhibits a favorable 12% likelihood of price increase, a 21% probability of reduction, and a 20% probability of no change in the near future. This information serves as a valuable tool for effective decision-making in the bank's daily management. Similarly, Fidelity Bank, PLC demonstrates a 10% probability of price increase, a 23% probability of reduction, and a 22% probability of no change in the near future, providing crucial insights for decision-making in the bank's day-to-day operations. Additionally, supplementary controllability results confirm the non-singularity of the controllability matrices for the share prices of both banks, indicating a creatively navigable financial market.

Nevertheless, there is a suggestion to explore an intriguing avenue for further investigation by integrating principal component analysis into the evaluation of the share prices of both banks.

**References**

1 Agwuegbo, S. O. N., Adewole, A. P. & Maduegbuna, A.N (2010). A random walk for stock market prices. *Journal of Mathematics and Statistics*,*6*(3),342-346.

2 Amadi, I. U, Igbudu R & Azor P. A.(2022). Stochastic analysis of the impact of growth rates on stock market prices. *Asian Journal of Economic, Business and Accounting.*

3. Adeosun, M. E., Edeki, S. O. & Ugbebor, O. O. (2015). Stochastic analysis of stock market price models:

 a case study of the Nigerian Stock Exchange(NSE*). WSEAS transactions on Mathematics, 14,* 353-363.

4. Davies, I. Amadi, I.U & Ndu, R.I(2019).Stability analysis of stochastic model for stock market prices. *International Journal of Mathematics and Computational Methods ,4,*79-86.

5. Ofomata, A. I. O.,Inyama, S. C.,Umana, R. A. & Omane,A.O(2017).A stochastic model of the dynamics of stock price for forecasting. *Journal of Advances in Mathematics and Computer Science*.*25(6*),1-24.

7 Ugbebor, O. O, Onah, S. E. & Ojowo, O.(2001). An empirical stochastic model of stock price changes. *Journal Nigerian Mathematical Society,20*,95-101

8 Osu, B. O., Okoroafor, A. C. & Olunkwa, C. (2009). Stability analysis of stochastic model of stock market price. *African journal of Mathematics and Computer Science 2(6),98-103*.

9 Mettle, F.O, Quaye, E.N.B & Laryea R.A(2014). A Methodology for stochastic Analysis of share prices as Markov chains with finite States.http://www.springerplus.com/content/3/1/057.

10. Bairagi A. & C, H. Kakaty S.(2015). Analysis of stock market price behavior: A markov chain approach. *International journal of Recent Scientific Research*, *6* (10), 7061-7066.

11 Zhang, D. & Zhang X. (2009). Study on forecasting the stock market trend based on stochastic analysis method. *International Journal of Business and Management*. 4(6). 163-170.

12 Eseoghene, J. I. (2011). The long run propect of stocks in the Nigeria capital Market: a aarkovian analysis*. Journal of Research in National Development (9)*1.

13 Christian, E. O. & Timothy, K. S. (2014). On predicting the long run behaviour of Nigerian bank stock prices: a Markov chain approach. *American Journal of Applied Mathematics and Statistics, 2(*4),212-215.

14 Osu, B. O., Emenyonu, S. C ., Ogbogbo, C. P. & Olunkwa, C. (2019). Markov models on share price Movements in Nigeria Stock Market Capitalization, *Applied Mathematics and Information Sciences an International Journal, 2*, 1-9.

15 Davies, I., Amadi, I.U., Amadi, C.P., Royal, C.A & Nanaka, S. O. (2023).Stability and controllability analysis of stochastic model for stock market prices, *International Journal of Statistics and Applied Mathematics,*8(4), 55-62.

16. Bullo, F., Lewis, A. D. & Lynah, K. M. (2002). Controllable kinematic reductions for mechanical systems: concept, computational tools, and examples. *National Science Foundation grants* 11S – 0118146 and CMS – 0100162.

17. Davies, I. (2005). Relative controllability of non-linear systems with delays in state and control. *Journal of the Nigerian Association of Mathematical Physics. 9,* 239 – 246.

18. Davies, I. (2006). Euclidean null controllability of linear systems with delays in state and control. *Journal of the Nigerian Association of Mathematical Physics. 10*. 553 – 558.

19. Manikonda, V. & Krishnaprasad, P. S. (2002).Controllability of a class of under-actuated mechanical systems with symmetry. *Automatical 38*, 1837 – 1850.

20. Matyukhin, V. I. (2004). The controllability of non-holonomic mechanical systems with constrained controls. *Journal of Applied Mathematics and Mechanics*. *68*,(5), 675 – 690.

21. Matyukhin, V. I. (2005).Controllability of mechanical systems with allowance for the drive dynamics. *Automation and Remote Control, 66*(12), 1937 – 1952.

22. Matyukhin, V. I. & Pyatnitskii, E. S. (2004). Controllability of mechanical systems in the class of controls bounded together with their derivatives. *Automation and Remote Control 65(*8), 1187 – 1209.