Numerical Analysis of Magnetohydrodynamics of Casson nanofluid flow over a rotating disk from Mathematical Physics Perspective

Abstract: This study examines the key characteristics of Casson nanofluid flow across a rotating disk, incorporating the effects of Brownian motion and thermophoresis. Additionally, the analysis considers the impact of thermal radiation and the Soret effect. Given the limited thermal efficiency of traditional fluids, nanofluids have gained considerable attention in recent years due to their enhanced heat transfer capabilities, making them valuable in various industrial and engineering applications. The research is motivated by its broad relevance in technological and engineering fields. The governing equations for fluid flow are converted into a system of nonlinear ordinary differential equations (ODEs) using appropriate similarity transformations. A numerical solution for these nonlinear ODEs is obtained through the numerical method via Shooting method combined with six order Runge Kutta Scheme. Maple software is used for the simulation. Graphical representations demonstrate how dimensionless physical parameters influence velocity, temperature, and concentration profiles. The findings reveal that thermal slip reduces fluid velocity, while an increase in the velocity slip factor leads to a decline in temperature distribution.

1.0 **Introduction**

In recent years, the enhancement of heat transmission and thermal conductivity has become a major focus for researchers. As technology continues to evolve, efficient heat transfer plays a crucial role in various industries, including transportation, manufacturing, and thermal power generation. However, a key challenge in designing heat transfer systems is the low thermal efficiency of conventional working fluids. To address this issue, scientists have explored innovative methods to improve thermal conductivity, with nanofluids—working fluids infused with nanoparticles—emerging as a promising solution. Research has demonstrated that nanofluids exhibit superior thermal properties compared to traditional fluids. The concept of nanofluids was first introduced by Choi (1995), sparking significant interest in their potential applications. Recent studies, such as those by Bacha *et al*. (2024), have reviewed the heat transfer enhancement capabilities of nanoliquids. Rafique *et al*. (2024) conducted numerical analyses on energy and mass transfer in nanofluid flows, while Kumar et al. (2021) investigated the stability of hybrid nanofluids under thermal radiation effects. Additionally, Waqas *et al*. (2021) found that hybrid nanofluids improve heat transfer efficiency in flat tubes, and Song *et al*. (2021) examined heat transport in nanofluids containing diverse nanoparticles.

The study of Casson fluid flow over a rotating disk has gained attention in fluid dynamics, particularly due to its non-Newtonian behavior. Casson fluids exhibit unique properties—such as high yield stress, causing them to behave like solids under low stress and liquids under high stress—making them valuable in biomedical engineering, food processing, and industrial applications.

Several researchers have contributed to this field: Jawad *et al*. (2021) compared the electrical conductivity and radiative properties of Casson nanofluids. Ali *et al*. (2024) numerically analyzed heat and mass transfer in Casson nanoliquid flows. Jyothi *et al*. (2021) explored Marangoni flow in Casson nanofluids with heat generation/absorption and chemical reactions.

Saeed et al. (2019) studied convective boundary effects using Laplace transforms, while Khan *et al*. (2020) examined porous media flow between deformable disks. Further investigations by Shah et al. (2019, 2018, 2018, 2019, 2019, 2019) focused on magnetohydrodynamic (MHD) heat transfer in nanofluids with microstructural variations. Khan *et al*. (2021) analyzed Maxwell nanofluids with thermal radiation and activation energy effects, whereas Waqas et al. (2021) studied time-dependent MHD flow in Oldroyd-B fluids with heat generation. Nadeem *et al*. (2021) investigated thermal radiation effects on stretched surfaces, and Makinde *et al*. (2018) analyzed viscosity variations in magnetized nanofluids. Hafeez *et al*. (2020) examined Soret and Dufour effects in porous media flows. Mustafa M. MHD nanofluid flow over a rotating disk with partial slip effects: Buongiorno model.

This work extends extend Khan *et el.* (2021) and Nadeem (2021) from two dimensional to the three-dimensional boundary layer flow of Casson nanofluid over a rotating disk, incorporating thermal radiation and Soret effects in the mass transfer equation. The numerical method method is employed for numerical solutions, and graphical analyses are presented to illustrate the impacts of key parameters—such as magnetic fields, thermal slip, thermophoresis, and Brownian motion—on velocity, temperature, and concentration profiles.

2.0 **Problem formulation**

The study examines the flow of Casson nanoliquid over a rotating disk. A cylindrical coordinate system (*r*, *ϕ*, *z*) is employed for the analysis, with the disk situated at *z*=0. The region *z*>0 is occupied by an electrically conducting, incompressible nanofluid. Slip effects are also incorporated into the model. A uniform magnetic field of strength *B*0​ is applied in the axial direction. Due to the low magnetic Reynolds number, the induced magnetic field is neglected. The wall temperature *Tw*​ is maintained higher than the ambient temperature *T*∞​ (i.e., *Tw*​>*T*∞​). Given the axial symmetry of the problem, all derivatives with respect to *ϕ* are disregarded.

The governing equations for this flow configuration are as follows:

 (1)

 (2)

 (3)

 (4)

 (5)

 (6)

The boundary condition attached to the governing equations above is

 (7)

Under the Rosseland approximation, the radiative heat flux (qᵣ) can be expressed as

 (8)

Here, **σ\*** represents the Stefan-Boltzmann constant, and **k\*** denotes the mean absorption coefficient. By expanding **T⁴** in a Taylor series around **T∞** and neglecting higher-order terms, we obtain:

 (9)

Our analysis focuses on the set of similarity transformations outlined below

 (10)

With the help of Eq. (10), the equations (2), (3), (5), and (6) are transformed into the following

 (11)

 (12)

 (13)

 (14)

The boundary settings are converted into

 (15)

Where

 (16)

In this context, the tangential stress  and radial stress  are respectively expressed as

 (17)

On the other hand, the local skin friction coefficient  and Nussselt are respectively transform to.

 (18)

 (19)

**3.0 Numerical Solution**

The system of strongly nonlinear partial differential equations—given by Equations (11) to equation (14)—along with the boundary conditions specified in Equation (15), is solved numerically. A shooting method integrated with a sixth-order Runge-Kutta scheme is applied to investigate the behavior of the system across a range of moderate flow, heat, and mass transfer parameters. To improve the precision of initial estimates and guarantee that the far-field boundary conditions are met, the Broyden quasi-Newton method is incorporated. The computational process is implemented and executed using Maple software for numerical simulation. The results obtained through Maple simulations are validated by comparing with existing results in literature and the result show excellent agreement

**4.0 Results and discussion**



 Fig 1: Impact of M on  Fig 2: Impact of  on 



 Fig 3: Impact of  on  Fig 4: Impact of  on 

The coupled nonlinear ordinary differential equations (11–14) under boundary conditions (15) were solved numerically, and the effects of key parameters—including the magnetic field (**M**), Prandtl number (**Pr**), Soret number (**Sr**), Brownian motion (**Nb**), radiation (**N**), Soret factor (**Sr**), and thermophoresis (**Nt**)—were analyzed. To validate the results, comparisons with published data (Table 1) showed strong agreement, while variations in energy transfer rates



 Fig 5: Impact of  on  Fig 6: Impact of  on 

 

 Fig 7: Impact of  on  Fig 8: Impact of  on 

under different conditions were also examined (Table 2). The study revealed that thermophoresis enhances energy transfer due to deeper nanoparticle penetration, which thickens the thermal boundary layer, whereas an increase in the thermal slip coefficient (**α**) reduces the local Nusselt number. Additionally, higher **Pr** values correspond to lower thermal diffusivity, leading to a steeper thermal gradient near the wall and a slightly thicker boundary layer compared to fluids with lower **Pr**.

 

 Fig 9: Impact of  on  Fig 10: Impact of  on 



 Fig 11: Impact of  on 

The influence of the magnetic field (**M**) on velocity was investigated in Fig. 1, showing that Lorentz forces reduce radial velocity, while Fig. 2 demonstrates that azimuthal velocity decreases with increasing thermal slip (**α**). Fig. 3 further confirms an inverse relationship between azimuthal velocity and **M**, whereas Fig. 4 illustrates that higher **Pr** values result in weaker temperature profiles. The impact of thermophoresis (**Nt**) on temperature distribution was examined in Fig. 5, revealing stronger thermal gradients at higher **Nt**, while Fig. 6 shows that Brownian motion (**Nb**) elevates the temperature profile **θ(η)**. Meanwhile, Fig. 7 indicates that temperature decreases with increasing **γ**, and Fig. 8 highlights how **Nt** expands the concentration distribution.

Table 1: Validation of Present result with existing result in literature

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Khuarm *et al*. (2025) | Present Result |
| M |  |  |  |  |  |  |  |
| 0 | 0.25 | 0.0648 | 0.2595 | -0.4167 | 0.0649 | 0.2596 | -0.4167 |
| 0.2 | 0.25 | 0.0477 | 0.1911 | -0.5095 | 0.0478 | 0.1912 | -0.5096 |
| 0.4 | 0.25 | 0.0365 | 0.1460 | -0.5995 | 0.0365 | 0.1461 | -0.5993 |
| 0.6 | 0.25 | 0.0291 | 0.1166 | -0.6803 | 0.0290 | 0.1167 | -0.6804 |
| 0.8 | 0.25 | 0.0241 | 0.0967 | -0.7515 | 0.0241 | 0.0967 | -0.7515 |
| 0.5 | 0 | 0 | 0.2186 | -0.7559 | 0 | 0.2186 | -0.7559 |
| 0.5 | 0.2 | 0.0286 | 0.1430 | -0.6621 | 0.0286 | 0.1430 | -0.6620 |
| 0.5 | 0.2 | 0.0396 | 0.0991 | -0.5845 | 0.0397 | 0.0991 | -0.5846 |
| 0.5 | 0.6 | 0.0431 | 0.0719 | -0.5220 | 0.0432 | 0.0719 | -0.5221 |

Table 2: Computational value of  against Nb, Nt, Pr, Sc, M, ,,

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Nb | Nt | Pr | Sc | M |  |  |  |  |
| 0.2 | 0.2 | 7.3 | 6.0 | 0.2 | 0.2 | 0.2 | 0.6 | 0.9864 |
| 0.4 | 0.2 | 7.3 | 6.0 | 0.2 | 0.2 | 0.2 | 0.6 | 0.8543 |
| 0.2 | 0,4 | 7.3 | 6.0 | 0.2 | 0.2 | 0.2 | 0.6 | 0.9763 |
| 0.2 | 0.2 | 9.0 | 6.0 | 0.2 | 0.2 | 0.2 | 0.6 | 0.8421 |
| 0.2 | 0.2 | 7.3 | 9 | 0.2 | 0.2 | 0.2 | 0.6 | 0.8144 |
| 0.2 | 0.2 | 7.3 | 6.0 | 0.6 | 0.2 | 0.2 | 0.6 | 0.7531 |
| 0.2 | 0.2 | 7.3 | 6.0 | 0.2 | 0.6 | 0.2 | 0.6 | 0.7994 |
| 0.2 | 0.2 | 7.3 | 6.0 | 0.2 | 0.2 | 1.5 | 0.6 | 0.9421 |
| 0.2 | 0.2 | 7.3 | 6.0 | 0.2 | 0.2 | 0.2 | 2.6 | 0.8901 |

The concentration profile **ϕ(η)** was found to decrease with higher **Nb** (Fig. 9), exhibiting an inverse relationship, while Fig. 10 demonstrates that larger Soret numbers (**Sr**) suppress concentration. Finally, Fig. 11 examines the effect of radiation (**N**), showing that increased radiative heat generation raises fluid temperature by accelerating particle motion and enhancing thermal energy.

**Conclusions**

This numerical study examines the influence of Soret and multiple slip effects on magnetized Casson nanofluid flow over a rotating disk. Given its relevance to engineering and industrial applications, this research holds significant practical value. The flow model was initially developed, and the governing partial differential equations (PDEs) were transformed into nonlinear ordinary differential equations (ODEs) using similarity variables. The numerical method was employed to obtain numerical solutions. Key findings from this investigation include:

**i.Velocity reduction**: The Casson parameter suppresses fluid velocity.
**ii.Thermal enhancement**: Temperature rises with increased thermal radiation.
**iii. Heat transfer improvement:** The energy transmission rate escalates with higher Casson factor values.
iv.**Magnetic damping:** Stronger magnetic effects decelerate fluid motion.
v.**Mass transfer impact**: A higher Schmidt number boosts the energy flow rate.

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