**ON WπGR- CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES: A MATHEMATICAL APPROACH**

**ABSTRACT:**

In this paper we introduce a new type of function called weakly π- generalized regular continuous functions . Also we study some characteristics and basic properties of wπgr-continuous functions. Moreover we study wπgr irresolute functions in topological spaces. The aim of this paper is to introduce the notion of weakly π- generalized regular continuous functions and weakly π- generalized regular irresolute functions in topological spaces.

**KEY WORDS**: wπgr-cl(A), wπgr-int(A), wπgr-continuous, wπgr-irresolute functions.

**1.INTRODUCTION**

Dontchev.J and Noiri.T [5] “introduced the concept of πg-closed set in topological space” .Levine [13] “initiated the study of generalized closed set(briefly g-closed set).The notion has been studied by many topologists, because g-closed sets are not only the generalization of closed sets. More importantly, they also suggested several new properties of topological spaces”. Later on N.Palaniappan[15] “studied the concept of regular generalized closed set in topological space”. Hussain(1966) [10] ,M.K.Singal and A.R.Singal(1968)[18] introduced “the concept of almost continuity in topological spaces”. K.Balachandran , P.Sundram and Maki [ 3] studied “generalized continuous maps in topological spaces”. Jeyanthi.Vand Janaki.C [12] “introduced and studied the properties of πgr-closed sets in topological spaces”. Levine introduced the concept of semi-continuous function in 1963. In 1980, Jain introduced totally continuous functions. In 1995, T. M. Nour introduced the concept of totally semi-continuous functions as a generalization of totally continuous functions. In 2011, S.S. Benchalli and Umadevi I Neeli introduced the concept of semi-totally continuous functions in topological spaces.

The purpose of this paper is to study wπgr-closure, wπgr -interior, wπgr-continuous functions.

**2. Preliminaries**

 Throughout this paper, X and Y denote the topological spaces (X,τ) and (Y,σ) respectively, on which no separation axioms are assumed. Let us recall the following definitions which are useful in the sequal.

**Definition :2.1**

A subset A of a topological space X is said to be

1. a regular open if A = int(cl(A)) and regular closed if A = cl(int(A))
2. (ii) π- open if A is the finite union of regular open sets and the complement of π- open is π- closed set in X.

The family of all open sets [regular open, π-open] sets of X will be denoted by O(X)(resp. RO(X), πO(X)]

**Definition :2.2**

A subset A of topological space X is said to be

 (1) a generalized closed set [13] (g-closed set) if cl (A) ⊂ U whenever A⊂ U and U ∈ O(X).

(2) a regular generalized closed [15] (briefly rg-closed set ) if cl(A)⊂U whenever A⊂ U and U∈ RO(X).

(3) a generalized pre regular closed set (briefly gpr -closed set) [8] if pcl (A) ⊂U whenever A ⊂ U and U ∈ RO(X) .

(4) a π-generalized closed (briefly πg- closed set) if cl (A) ⊂ U whenever A ⊂ U and U ∈ πO(X).

(5) a πgα - closed set[11] if αcl (A) ⊂ U whenever A⊂U and U∈ πO(X).

(6)a π\*g-closed set[7] if cl(int(A)) ⊂ U whenever A⊂U and U∈ πO(X).

(7) a πgb-closed set [20] if bcl(A) ⊂ U whenever A⊂U and U∈ πO(X).

(8) a πgp-closed set [17] if pcl(A) ⊂ U whenever A⊂ U and U∈ πO(X).

(9) a wπgr- closed set if cl( int A)  U whenever AU and U is πgr-open in X.

Here in this chapter the concept of continuity via Wπgr-closed set is introduced and obtained few of its properties.

Also, the existence definitions if continuity of various closed sets are defined and the same is used to compare with the new class of continuous maps called Wπgr-continuity.

**Definition 2.3:**

A map f : XY is said to be

(1) a continuous function if f(V) is closed in X for every closed set V in Y.

(2) a w- continuous function if f(V) is w -closed in X for every closed set V in Y.

(3) a rg- continuous function if f(V) is rg- closed in X for every closed set V in Y

(4) a π -continuous function if f(V) is π- closed in X for every closed set V in Y

(5) a πg- continuous function if f(V) is πg -closed in X for every closed set V in Y

(6) a g- continuous function if f(V) is g -closed in X for e very closed set V in Y.

(7) a wg -continuous function if f(V) is wg- closed in X for every closed set V in Y.

(8) a πgα continuous function if f(V) is πgα -closed in X for every closed set V in Y.

(9) a πgp - continuous function if f(V) is πgp -closed in X for every closed set V in Y.

(10) a π\*g- continuous function if f(V) is π\*g -closed in X for every closed set V in Y.

11) a rwg continuous function if f(V) is rwg- closed in X for every closed set V in Y.

11) a wπgr continuous function if f(V) is wπgr - closed in X for every closed set V in Y.

**3. wπgr –closure and interior**

**Definition 3.1:**

For any set A⊆X, the wπgr- closure of A is defined as the intersection of wπgr- closed sets containing A.

The complement of wπgr- closure is the wπgr - interior.

We write wπgr-cl(A)=∩{F:AF is wπgr closed in X

**Definition 3.2:**

A function f: (X , τ )  (Y , ϭ ) is called wπgr - continuous if every f)V) is wπgr -closed in X for every closed set V of Y.

**Example 3.3:**

Let X = { a , b ,c ,d }

τ = { φ ,{a} , {b},{a,b} , { a , b , c } , X }

τᶜ = { x , { b , c , d } , { a,c,d } , { c,d } , { d }, φ }

wπgr – closed sets = { X , { c } ,{ d} , { c,d } ,{ a ,c , d } ,{ b , c , d } , φ }

Let Y = { a , b ,c ,d }

ϭ = { φ , { a } , { c} , { a , c }, Y}

ϭᶜ = { φ , { b , d }, { a , b , d }, { b , c , d }, Y }

Define f: (X , τ )  (Y , ϭ ) by f(a) = b , f(b ) = c , f(c ) = a, f(d ) = d .

f(a) =c , f(b) =a , f(c) =b , f(d) =d

Here the inverse image of the closed sets in Y are wπgr -closed in X . Hence the function f is wπgr continuous.

**Example 3.4:**

X = { a , b , c , d }

τ = { φ , { a } ,{ d } , { a ,d } ,{ b ,d } ,{ a , b , d } , X}

τᶜ = { X , { b ,c , d } ,{ a , b , c } ,{ b , c } , { a , c } , { c } , φ }

wπgr-closed sets = { φ , { b } , { c } , { a , c } , { b , c } , { c , d } , { a , b , c } , { a , c , d } , { b , c ,d } ,X }

w – closed sets **=** { φ , { c } , { a , c } , { b , c } , { a , b , c } , { b , c ,d } ,X }

πgr -closed sets **=**{φ ,{ c }, { a , c } , { b , c } , { c , d } , { a , b , c } , { a , c , d } , { b , c ,d } ,X }

πg- closed sets={φ , { c } , { a , c } , { b , c } , { c , d } , { a , b , c } , { a , c , d } , { b , c ,d } ,X }

rg-closed sets= { φ , { a , b } , { c } , { a , c } , { b , c } , { c , d } , { a , d } , { a , b , d } , { a , b , c } , { a , c , d } , { b , c ,d } ,X }

Let Y = { a , b ,c ,d }

 ϭ= { φ , { a , c , d }, Y}

ϭᶜ = { φ , { b } , Y }

Define f: (X , τ )  (Y , ϭ ) by f(a) = b , f(b ) = c , f(c ) = a, f(d ) = d .

f(a) =c , f(b) =a , f(c) =b , f(d) =d

Hence the inverse image of closed set in Y is wπgr -closed , but not w- closed, πg-closed, πgr- closed, rg-closed . Hence wπgr -continuity need not be w - continuous, πg-continuous, πgr- continuous, rg- continuous .

**Example 3.5:**

Let X = { a , b ,c ,d }

τ = { φ ,{a} , {b},{a,b} , X }

τᶜ = { x , { b , c , d } , { a,c,d } , { c,d } , φ }

g - closed sets = { φ , {c}, {d},{a,c},{a,d},{b,c}, {b,d},{ c,d } , { a , b ,c }

{ b ,c ,d } ,{ a , b , d } , {a,c,d}, X }

wπgr – closed sets = { X , { c } ,{ d} , { c,d } ,{ a ,c , d } ,{ b , c , d } , φ }

wg- closed set = { X , { c } ,{ d} , { c,d } ,{ a ,c , d } ,{ b , c , d } , { a , c }, { a , d } ,{ b , c }, { b , d }, { a , b , c }, { a , b ,d }, { a ,c , d }, φ }

πgr- closed sets = { φ , {c}, {d},{a,c},{a,d},{b,c}, {b,d},{ c,d } , { a , b ,c }

{ b ,c ,d } ,{ a , b , d } , {a,c,d}, X }.

πgα - closed sets = { φ , {c}, {d},{a,c},{a,d},{b,c}, {b,d},{ c,d } , { a , b ,c }

{ b ,c ,d } ,{ a , b , d } , {a,c,d}, X }.

πgp - closed sets = { φ , {c}, {d},{a,c},{a,d},{b,c}, {b,d},{ c,d } , { a , b ,c }

{ b ,c ,d } ,{ a , b , d } , {a,c,d}, X }.

πg - closed sets = { φ , {c}, {d},{a,c},{a,d},{b,c}, {b,d},{ c,d } , { a , b ,c }

{ b ,c ,d } ,{ a , b , d } , {a,c,d}, X }.

rwg - closed sets = { φ , {c}, {d},{a,c},{a,d},{b,c}, {b,d},{ c,d } , { a , b ,c }

{ b ,c ,d } ,{ a , b , d } , {a,c,d}, X }.

ϭ = { φ , { b , d }, Y}

ϭᶜ = { φ , { a , c } , Y }

Define f: (X , τ )  (Y , ϭ ) by f(a) = b , f(b ) = c , f(c ) = a, f(d ) = d .

f(a) =c , f(b) =a , f(c) =b , f(d) =d

Thus f(a , c ) = { a , c } is πgr- closed in X , but not wπgr - closed in X .

Thus f(a , c ) = { a , c } is πgr –closed, wg- closed, πg- closed, g –closed, rwg –closed, πgα- closed, πgp -closed in X , but not wπgr-closed in X .

Hence wg -continuous,πg- continuous, πgr- continuous , rwg- continuous ,g- continuous , rwg- continuous , πgα- continuous, πgp- continuous need not be wπgr -continuous.

**4. Composition of Mappings**

**Definition 4.1:**

Let f : X Y ,g : Y  Z ,gf : X  Z .

f : X Y is defined as f (a) =a , g( a ) = a (gf ) (a) = g ( f (a)) =g ( a) = a

**Remark 4.2:**

Composition of two wπgr -continuous functions need not be wπgr -continuous.

**Example 4.3:**

Consider X = { a , b , c , d }

τ = { φ , { a } ,{ d } , { a ,d } ,{ b ,d } ,{ a , b , d } , X}

ϭ = { φ , { a , d } , { a , c , d }, Y}

= { φ , { c , d }, Y}

ᶜ = { φ , { a , b }, Y}

Here f: (X , τ )  (Y , ϭ ) and g : (Y , ϭ )  (Z , ἠ ) are identity mappings.

(gf ) (a ,b ) = { a , b } is not wπgr closed in X .

****Composition of two wπgr **-**continuous functions need not be wπgr - continuous .

**Definition 4.4:**

A space X is said to be wπgr -Tspace if every wπgr -closed set is closed.

**5.Wπgr**- **irresolute functions**

**Definition 5.1:**

A function f: (X , τ )  (Y , ϭ ) is called wπgr -irresolute if inverse image of V is wπgr- closed in X for every closed set V of Y.

**Theorem 5.2:**

Let f: (X , τ )  (Y , ϭ ) and g : (Y , ϭ )  (Z , ) be any two functions .Then

(i)(gf ) is wπgr- irresolute if g is continuous and f is wπgr-continuous

( ii) (gf ) is wπgr- continuous if g is wπgr- continuous and f is wπgr- irresolute.

( iii) (gf ) is wπgr -continuous if f is wπgr- irresolute and g is wπgr -continuous and is wπgr -

T.space.

**Proof:**

(i)Let V be closed in Z . Then g(v) is closed inY . Since g is continuous, wπgr continuous of f implies f (g(v)) is wπgr closed in X . (gf ) (v ) ) is wπgr closed in X.

 Hence (gf ) is wπgr continuous.

**(ii)** Let V be closed in Z . Since g is wπgr continuous g (v) is wπgr closed in Y. As f is wπgr irresolute f (g(v)) = (gf ) (v ) ) is wπgr closed in X

Hence (gf ) is wπgr continuous .

**(iii)** Let V be closed in Z . Since g is wπgr continuous g (v) is wπgr closed in Y. As Y is wπgr T.space. g(v) is closed inY

 Irresoluteness of f implies f (g(v)) = (gf ) (v ) ) is closed . Hence (gf ) (v ) ) is closed in Y and hence (gf ) is continuous .

**Remark 5.3:**

 The above relations are diagrammatically represented as follows.

 continuous

 rgcontinuous πgr continuous

πgα continuous

wg continuous Wπgrcontinuous w continuous

 πgcontinuous g - continuous

 πgp continuous

fig 1-Wπgr-continuous

 **CONCLUSION**

In this paper we have introduced WπGR -continuous andWπGR -irresolute functions in topological spaces and studied some of their basic properties. This study can be extended to fuzzy topological spaces and bitopological spaces.

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Details of the AI usage are given below:

1.

2.

3.

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