MATHEMATICAL ANALYSIS OF MALNUTRITION IN CRECHE

Abstract

Malnutrition is a major issue in emerging and poor nations, particularly in some African nations. Its effect in children under five years old can not be over emphasize. Our model establishes the existence and uniqueness of solutions to malnutrition model equations, and numerically, the population of children in each class was obtained by parameter values; converting the differential equations system into Laplace transform, and their inverse Laplace transform was achieved. The outcome of the study showed a significant rise in the number of malnourished children. Malnutrition incidence can be considerably decreased by effective prevention and control measures and the result from this study has a positive impact on public health thereby lowering the rates of morbidity and mortality in children less than five years of age.

Keywords: Malnutrition, Children, Nutrition, Model.

1 Introduction

Malnutrition is one of the world's biggest problems. It occurs in children when their bodies doesn't receive enough nutrients, thereby leading to negative health challenges. According to Partha and Nandita in [1] children, especially in rural areas are susceptible to malnutrition due to inadequate dietary intake, recurrent infections, lack of adequate care and unequal distribution of food within the family. Malnutrition, a major determinant of maternal and child health, has significant negative effects on children's brain and cognitive development [2]. According to a UNICEF report, malnutrition may be one of the leading causes of infant mortality worldwide. According to the 2016 UNICEF-WHO-World Bank [3], Joint Estimates of Child Malnutrition, globally 155 million and 52 million children under the age of five are stunted and wasted, respectively. In addition, 17 million children under the age of five were severely underweight. UNICEF studied the three key burdens of under-nutrition that continue to undermine children's ability to survive and grow, these burdens are stunting, fatigue, and obesity with greater percentage in Asia and Africa as show on table 1 [4].

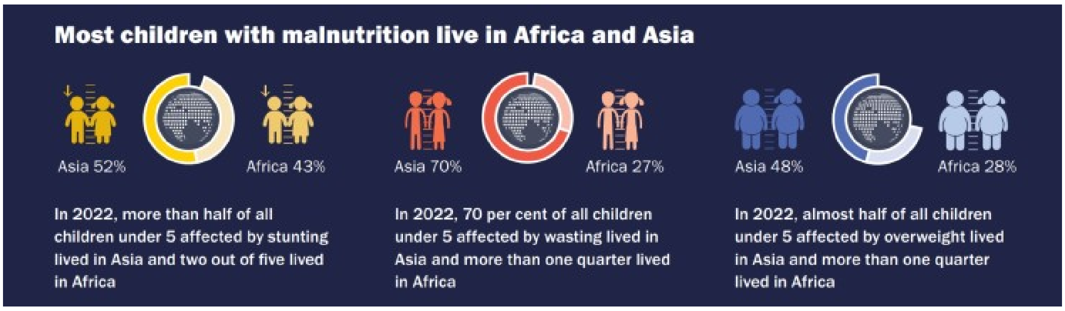

Figure 1: Percentage of children with malnutrition live in Africa and Asia [4]

Joint Malnutrition Assessment (JME) launched in 2023 to prevent all forms of food insecurity and stop malnutrition before it starts; children and their families need access to nutritious diet, essential services and positive experiences can get them going in the right direction, survive and thrive. Today an important way to maintain healthy eating habits is at risk many countries. The global food and nutrition crisis is deepening due to the ongoing ramifications of poverty, economic hardship (as can be seen in Nigeria and some other countries), conflict, climate change and the epidemic outbreak. Important urgent measures should be geared towards protecting food, especially for large numbers of mothers and children in areas affected and prepared for the future where the right to food becomes a reality for every child [5]. Malnutrition often affects children's growth. A healthy child between the ages of 0 and 5 years typically grows $2kg$ to $4kg$ and grows about $5cm$ to $8cm$ per year [6]. If a child grows less than $2inches(< 5.08cm)$ per year after the second birth, which results to a growth problem [7] while studies have shown that approximately one in five children suffer from some degree of chronic malnutrition [8].

In Nigeria, an estimated 5.93 million children under the age of five will suffer from acute malnutrition between May 2022 and April 2023, of which 1.6 million are severely malnourished and require treatment. Between May 2022 and April 2023, approximately 6 million infants and children aged 0-59 months in north-west and north-east Nigeria are expected to suffer from acute malnutrition. This includes $1,623,130$ cases of severe acute disability (SAM) and $4,308,404$ cases of moderate acute disability (MAI). In addition, $511,890$ pregnant and lactating women are severely malnourished and may require nutritional intervention and the map of Nigeria in 1 showed that Northern states have high prevalence of malnutrion [9].

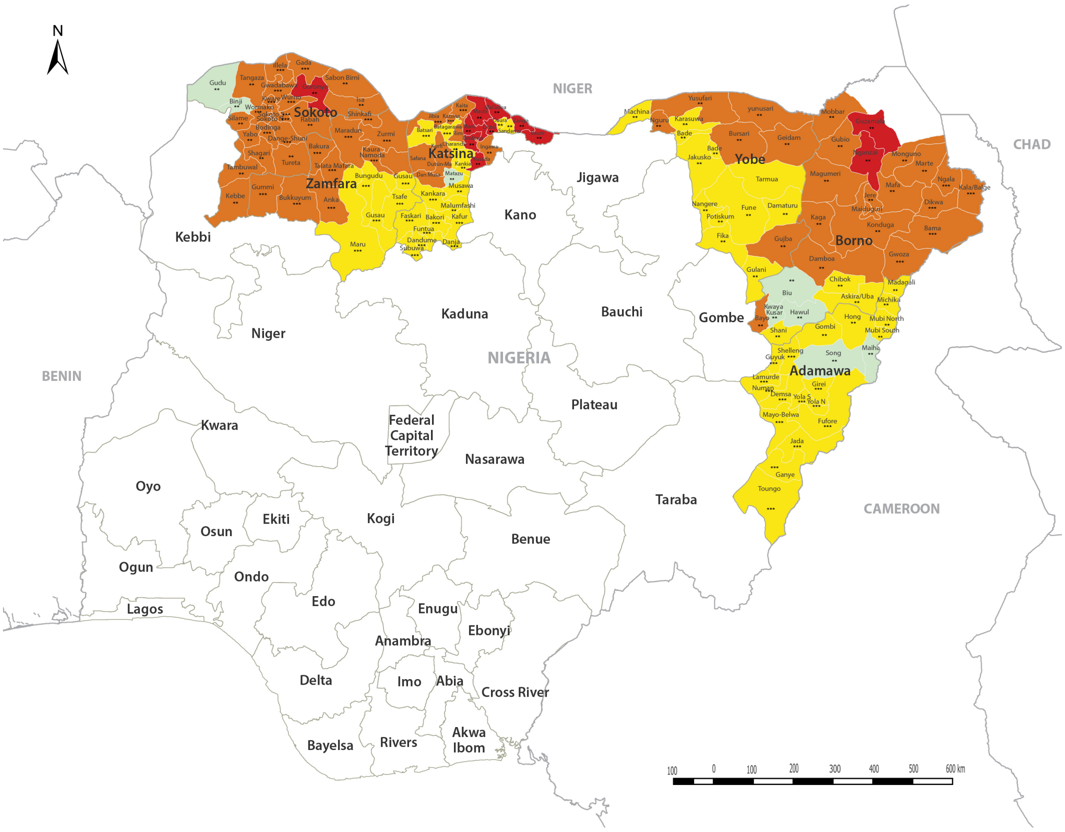

Figure 2: Projected Acute Malnutrition Situation — January - April 2023 [9]

This article covered examined the application of mathematical models and analysis to comprehend malnutrition and the possible measures to put in place in controlling it in children. This study also carried out thorough investigation of malnutrition by applying the application of mathematical model, in analyzing malnutrition and positive effect of the strategies proposed by UNICEF and WHO in compacting the growth of malnutrition in Africa.

2 Model Formulation

Let the P, C, C_N and C_M represent the total population of humans, population of children $0-18$ years, class of children under 5 suffering malnutrition and class of children under 5 free from malnutrition respectively. Let P be the entire population at a given time t. The group of children under 5 years of age grows by $\alpha_1 e^{r_1} P$ and declines, dividing it into healthy and malnourished classes. The non-malnourished class increased by $(1-\tau)\alpha_2 e^{r_2} C$ and the malnourished class recovered at $\alpha_3 C_M$. Finally, the section of the malnourished children increased by $\tau\alpha_2 e^{r_2} C$ and decreases with mortality (death induced by malnutrition) at ϕC_M and number of children recovered from malnutrition at the rate $\alpha_3 C_M$.

$$\left.\begin{array}{rcl} \dot{P} &=& \Lambda - \alpha_1 a^{r_1} P \\ \dot{C} &=& \alpha_1 a^{r_1} P - \alpha_2 a^{r_2} C \\ \dot{C_N} &=& (1-\tau)\alpha_2 a^{r_2} C + \alpha_3 C_M \\ \dot{C_M} &=& \tau\alpha_2 a^{r_2} C - (\alpha_3 + \phi) C_M \end{array}\right\} \tag{2.1}$$

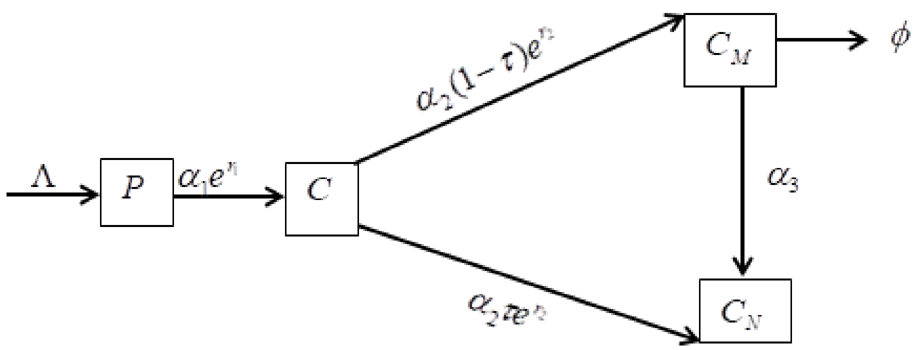

Figure 3: Model Diagram

Table 1: Meaning of parameters

Parameters	Description	Values	Reference
α_1	Extraction rate of children from the entire population	0.054	Assumed
α_2	Progression rate of children under 5 with malnutrition	0.0045	Assumed
r_1	Growth rate children from the whole population	0.067	[6]
r_2	Growth rate of children under 5 malnutrition	0.0508	[7]
τ	Fraction of children under 5 with malnutrition	0.2	[8]
ϕ	Malnutrition induced death rate	0.45	[10]
a	Euler's number	2.71828	
Λ	Total population at a given time t	232679478	[11]
α_3	Recovery rate of under 5 children	0.742	[12]

3 Existence and Uniqueness of the Model Equation

Theorem 3.1. *Let $R^4_+ \in \Omega$ denote the region of feasibility for the system (2.1). If the model equations (2.1) is continuous, then the existence and uniqueness of the state variables $(P, C, C_N, C_M) \in \Omega$ exist at any give time $t \geq 0$.*

Proof. From model equation (2.1)

$$\begin{aligned} \dot{P} &= \Lambda - \alpha_1 a^{r_1} P \\ u(t, P) = \frac{du}{dP} &= -\alpha_1 a^{r_1} \end{aligned}$$

Given that $u(t, P)$ and its derivatives is continuous. The application of Cauchy-Lipschitz condition will be employed to determine the existence and uniqueness of the model equation 2.1.

$$\begin{aligned} \mid u(t, P_1) - u(t, P_2) \mid &= \mid -\alpha_1 a^{r_1} P_1 + \alpha_1 a^{r_1} P_2 \mid \\ &= \mid -\alpha_1 a^{r_1} \mid\mid P_1 - P_2 \mid \\ &\leq \mid -1 \mid\mid \alpha_1 a^{r_1} \mid\mid P_1 - P_2 \mid \\ &\leq \mid \alpha_1 a^{r_1} \mid\mid P_1 - P_2 \mid \\ &\leq L \mid P_1 - P_2 \mid \end{aligned}$$

$L = \alpha_1 a^{r_1}$ denote the Lipschitz constant and applying the same argument, $u(t, C), u(t, C_N), u(t, C_M)$ together with their derivatives are continuous respectively. Hence, the model equations (2.1) satisfied the Lipschitz condition. Therefore, there exist a unique solution to the state variables (P, C, C_N, C_M) at any given time $t \geq 0$ □

By application of Laplace transform [13], equation 2.1 becomes

$$\left.\begin{aligned} \mathcal{L}\dot{P}(s) &= \Lambda - \alpha_1 a^{r_1} \mathcal{L}P(s) \\ \mathcal{L}\dot{C}(s) &= \alpha_1 a^{r_1} \mathcal{L}P(s) - \alpha_2 e^{r_2} \mathcal{L}C(s) \\ \mathcal{L}\dot{C_N}(s) &= (1 - \tau)\alpha_2 a^{r_2} \mathcal{L}C(s) + \alpha_3 \mathcal{L}C_M(s) \\ \mathcal{L}\dot{C_M}(s) &= \tau\alpha_2 a^{r_2} \mathcal{L}C(s) - (\alpha_3 + \phi)\mathcal{L}C_M(s) \end{aligned}\right\} \quad (3.1)$$

On simplification of 3.1, we have obtain the following equations below:

$$\left.\begin{aligned} sP(s) &= \Lambda - \alpha_1 a^{r_1} P(s) \\ sC(s) &= \alpha_1 a^{r_1} P(s) - \alpha_2 a^{r_2} C(s) \\ sC_N(s) &= (1 - \tau)\alpha_2 a^{r_2} C(s) + \alpha_3 C_M(s) \\ sC_M(s) &= \tau\alpha_2 a^{r_2} C(s) - (\alpha_3 + \phi) C_M(s) \end{aligned}\right\} \quad (3.2)$$

Factorizing each of the equations in equation 3.2

$$sP(s) = \Lambda - \alpha_1 a^{r_1} P(s)$$

$$P(s) = \frac{\Lambda}{s + \alpha_1 a^{r_1}} \tag{3.3}$$

Therefore, the application of inverse Laplace transform [11], yields equation 3.4

$$P(t) = \Lambda a^{-\alpha_1 a^{r_1} t} \tag{3.4}$$

Similarly, we have

$$sC(s) = \alpha_1 a^{r_1} P(s) - \alpha_2 a^{r_2} C(s) \tag{3.5}$$

Substituting equation 3.3 into equation 3.5, we have

$$C(s) = \frac{\Lambda \alpha_1 a^{r_1}}{(s + \alpha_1 a^{r_1})(s + \alpha_2 a^{r_2})} \tag{3.6}$$

Therefore, by resolving equation (3.6) into partial fraction and using the inverse Laplace transform, we have

$$C(t) = \frac{\Lambda \alpha_1 a^{r_1}}{\alpha_2 a^{r_2} - \alpha_1 a^{r_1}} \left(e^{-\alpha_1 a^{r_1} t} - e^{-\alpha_2 a^{r_2} t} \right) \tag{3.7}$$

Similarly, we have

$$\left.\begin{aligned} C_N(t) = {} & \Lambda(1-\tau) - \tfrac{\alpha_2 \Lambda (1-\tau) a^{r_2}}{(\alpha_2 a^{r_2} - \alpha_1 a^{r_1})} e^{-\alpha_1 a^{r_1} t} - \tfrac{\alpha_1 \Lambda (1-\tau) a^{r_1}}{(\alpha_1 e^{r_1} - \alpha_2 a^{r_2})} e^{-\alpha_2 a^{r_2} t} + \alpha_1 \alpha_2 \alpha_3 \Lambda \tau a^{(r_1+r_2)} \\ & \left(\tfrac{e^{-\alpha_1 a^{r_1} t}}{(\alpha_2 a^{r_2} - \alpha_1 a^{r_1})(k - \alpha_1 a^{r_1})} + \tfrac{e^{-\alpha_2 a^{r_2} t}}{(\alpha_1 a^{r_1} - \alpha_2 a^{r_2})(k - \alpha_2 a^{r_2})} + \tfrac{e^{-kt}}{(\alpha_1 a^{r_1} - k)(\alpha_2 a^{r_2} - k)} \right) \end{aligned}\right\} \tag{3.8}$$

$$\left.\begin{aligned} C_M(t) = {} & \alpha_1 \alpha_2 \Lambda a^{r_1+r_2} \left(\tfrac{e^{-\alpha_1 a^{r_1}} t}{(\alpha_2 a^{r_2} - \alpha_1 a^{r_1})((\alpha_3+\phi) - \alpha_1 a^{r_1})} + \tfrac{e^{-\alpha_2 a^{r_2}} t}{(\alpha_2 a^{r_2} - \alpha_1 a^{r_1})((\alpha_3+\phi) + \alpha_1 a^{r_1})} \right. \\ & \left. + \tfrac{e^{-kt}}{(\alpha_1 a^{r_1} - (\alpha_3+\phi))(\alpha_2 a^{r_2} - (\alpha_3+\phi))} \right) \end{aligned}\right\} \tag{3.9}$$

Where $k = (\alpha_3 + \phi)$, let $N(t) = P(t) + C(t) + C_N(t) + C_M(t)$.

In this study, the population under study is considered constant owning to the following assumptions

1. Birth and death rates are constant
2. Yearly increasing rate of adults and children migrate of Nigeria

To show that the population under study is constant, the theorem below is considered

Theorem 3.2. *Let $N(t)$ be constant on the interval $[t_i, t_{i+1}]$, then $N(t)$ is continuous on $[t_i, t_{i+1}]$.*

Proof. Given that $N(t)$ is a constant function on $[t_i, t_{i+1}]$ and let c be the constant value of the function. It suffices to show that $N(t)$ is continuous on $[t_i, t_{i+1}]$.
For any $t \in [t_i, t_{i+1}]$ then $N(t) = c$. $N(t)$ is continuous at a point $t = x$

$$lim_{x \longrightarrow a} N(t) = N(a)$$

Since $N(t) = c$ for all $t \in [t_i, t_{i+1}]$, Then $lim_{x \longrightarrow a} N(t) = lim_{x \longrightarrow a} c = c = N(a)$. Therefore, the population $N(t)$ is continuous at every time $t \in [t_i, t_{i+1}]$. Since $N(t)$ is continuous at in the interval of time $[t_i, t_{i+1}]$, thus $N(t)$ is continuous on the interval $[t_i, t_{i+1}]$. □

Using the the values in table 1, the values of $P(t)$, $C(t)$, $C_N(t)$ and $C_M(t)$ were recorded from 2024 to 2029

Table 2: Prediction in the population of children

Year	time (t)	$C(t)$	$C_N(t)$	$C_M(t)$	% $C_N(t)$
2024	1	15543558	27248	3867	0.18%
2025	2	29961557	135810	15140	0.45%
2026	3	43329852	305459	27335	0.71%
2027	4	55719215	530833	39170	0.95%
2028	5	67195673	807560	50298	1.2%

4 Discussion of Result

The table above showed the annual progression in the population of children i.e children under 5 years of age, children under 5 years of age suffering malnutrition, and children under 5 years of age free from malnutrition respectively. Therefore, progressive rise in percentage of children with malnutrition calls for the noble intervention of WHO, UNICEF, the SDGs and Federal government of Nigeria.

5 Conclusion

To curb the menace of malnutrition in children, Nigeria government and humanitarian aid that saves lives should do well to increase household food coverage in conjunction with supplemental nutrition treatments to address the nutritional needs of children and internally displaced people (IDPs) and increase food consumption among them. it is critical to address the underlying causes of these conditions and strengthen the bonds that bind Nigeria government, humanitarian and development initiatives to fight hunger and malnutrition together. The measures below when adopted will drastically limit the number of C_M while C_N grows more exponentially.

1. Putting in place community-based nutrition education initiatives to raise awareness of the value of a healthy, balanced diet.

2. Increase the availability of reasonably priced, nutrient-dense food by implementing programs like food stamps, community gardens, and food subsidies.

3. Increasing and varying agricultural output in order to lessen reliance on imports and guarantee a sustainable food supply.

4. Expanding the availability of nutrition services and health infrastructure, such as routine physical examinations and nutritional status monitoring.

5. Encourage breastfeeding and provide babies and young children with appropriate food to prevent early malnutrition.

6. Reinforce safety nets and social protection initiatives to assist marginalized groups and guarantee food access during emergencies or bad times.

Abbreviation

WHO: World Health Organization
SDGs: Sustainable Development Goals
UNICEF: United Nations International Children Emergence Fund.

Conflict of Interest

The author declared there is no conflict of interest.

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