Linear Programming for Resource Allocation and Profit Maximization in Furniture Production

Abstract

This case study explores the use of linear programming, specifically the simplex method, to optimize the production process in a furniture manufacturing company. The main objective is to maximize profits by identifying the most effective production mix of items such as dining tables, chairs, sofa sets, dressing tables, shoe racks, and beds, all while adhering to constraints on materials like wood, fabric, paint, Sun mica, stuffing, accessories, and labor costs.

The study utilizes Excel Solver to apply linear programming and solve the problem of profit maximization. By entering the production objectives and constraints into Solver, it computes the best possible production quantities for each product, ensuring maximum profitability while staying within resource limits.

The findings reveal the optimal number of units to produce for each item, providing valuable insights for businesses on how to efficiently allocate resources and achieve financial targets. Overall, the case study highlights the power of linear programming, facilitated by Excel Solver, in helping businesses make informed decisions that lead to increased profit margins in environments with limited resources.

Keywords: Linear programming, Simplex method, Decision variables, Optimization, Profit maximization, Excel.

Introduction:

A linear objective function subject to linear constraints can be maximized or minimized using linear programming, a potent mathematical technique for optimization issues.

The concept was developed by George B. Dantzig in 1947, and it has since become integral to operations research and economics, especially in resource allocation and scheduling.

One of the significant advantages of LP is its adaptability, as exemplified by the Simplex algorithm, which efficiently solves linear programming problems even in complex scenarios and extensive applications

In this study, we apply linear programming for furniture production, where the focus is on determining the optimal quantities of dining tables, chairs, sofa sets, dressing tables, shoe racks and beds to produce for maximum profit. The furniture production operates under various constraints, including demand, material quantity & labor availability.

General Form of a Linear Programming Model

The linear programming problem is generally expressed as:

Maximize (or Minimize) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ (objective function) …(1.1)

Subject to the constraints:

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (<,=,>) \ b_1$

$a_{21}x1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (<,=,>) \ b_2$

…. …. …. …. ….

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (<,=,>) b_m$ … (1.2)

And non-negative restrictions: $x_j \geq 0$ for $j = 1, 2, \ldots, n$

Where a_{ij}'s, bi, s, and c_j's are constants and x_j's are variables.

Any of the 3 indications <, = and > may be present in the circumstances specified by (1.2). The standard form of a linear programming problem for the simplex technique is as follows:

(a) All constraints are expressed as equations using excess and slack variables.

(b) For each constraints all $b_i > 0$, if any b_i is negative then multiply the corresponding constraint by − 1.

(c) Always remember, the problem must be of maximization type, if not, convert it in maximization type by multiplying the objective function by −1.

This is one way to formulate the linear programming problem of n variables and m constraints using slack and excess variables:

Optimize

$$Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n + 0.s_1 + 0.s_2 + \dots + 0.s_m \text{ (Objective Function)} \quad \dots(1.3)$$

Subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + s_1 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_2n_{xn} + s_2 = b_2$$

$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$

$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + s_m = b_m \quad (1.4)$$

and non – negative restrictions

$$x_j > 0,\ s_i > 0,\ j = 1, 2\dots n,\ i = 1, 2\dots, m$$

Where a_{ij}'s, b_i's and c_j's are constants and x_j's and s_i's are variables

Review of Literature

Linear programming (LP) has been extensively used in optimization problems across industries. Dantzig's pioneering work on the Simplex Method laid the foundation for LP applications in real-world resource allocation and profit maximization. Research by Sherali and Yao (2007) highlighted the role of LP in supply chain design for profit maximization, while Patidar and Choudhary emphasized its utility in small-scale industries for optimizing resource use. The application of LP in furniture production remains underexplored but offers significant potential, as suggested by Schulze's works on operational efficiency. Studies have demonstrated that LP not only aids decision-making but also ensures sustainable use of resources.

The application of Linear Programming (LP) in manufacturing has been a significant focus of research, especially concerning its use in profit maximization and resource optimization. This literature review highlights key studies related to the application of linear programming in production optimization, particularly in the furniture manufacturing industry, and the role of Excel Solver in solving LP problems.

Taha (2017) explains the fundamental principles of linear programming and its applications in business, especially for optimizing manufacturing processes. He discusses how LP helps businesses allocate resources efficiently, leading to cost savings and maximized profits. Linear programming has been successfully applied in industries such as textiles, automotive, and furniture manufacturing to optimize production and profit.

Charnes and Cooper (1950) laid the foundation for LP's role in business and industry by demonstrating how it can be used to allocate resources effectively and maximize profit. Their seminal work set the stage for further research in operational research and the practical applications of LP.

Gass (2005) highlighted the role of LP in production optimization and cost reduction across various industries. His work emphasized how LP allows businesses to determine the most efficient product mix while considering the constraints of limited resources. In the context of furniture production, LP is used to determine optimal quantities of items such as tables, chairs, and sofas to maximize profit while respecting material and labor constraints.

Winston (2004) further examined LP's role in decision-making processes in industries where profit maximization is a primary goal. His research underscores the importance of formulating LP models with clear objectives (profit maximization) and constraints (materials, labor costs) to derive optimal production strategies.

According to Gurobi (2018), Excel Solver has become a popular tool in businesses due to its simplicity, cost-effectiveness, and ability to handle LP problems of moderate complexity. The software enables users to input constraints and objective functions and quickly find the optimal solution to the problem at hand.

Meyer (2014) supports this, stating that Excel Solver is particularly effective in small to medium-sized enterprises that lack access to more complex optimization software. Meyer’s study found that Solver could handle multiple constraints and objectives, making it well-suited for production optimization problems in industries such as furniture manufacturing.

Srinivas and Rao (2019) demonstrated the use of LP in optimizing the production mix of different furniture products, including tables, chairs, and sofas, while considering constraints such as raw materials, labor, and production capacity. They concluded that LP models can lead to improved production efficiency and higher profits by providing optimal production schedules. Soni and Jain (2020) focused on applying LP in the furniture industry to minimize material wastage and optimize resource allocation. Their study showed that LP could effectively reduce production costs by ensuring that resources were used efficiently, thus contributing to higher profit margins.

The literature reviewed demonstrates that linear programming is a powerful tool for solving optimization problems in production, particularly for maximizing profit in manufacturing settings. Excel Solver, as an accessible tool for solving LP problems, has made it easier for small and medium-sized businesses to optimize their operations. Studies in the furniture manufacturing sector have shown that LP models are highly effective in determining the optimal production mix and achieving cost efficiencies. The growing body of literature suggests that linear programming will continue to be an essential decision-making tool for businesses aiming to optimize their production processes and increase profitability.

Research Methodology

This study employs linear programming to develop an optimal production plan for a furniture shop. Using the Simplex Method, the problem is modeled with an objective function to maximize profit while considering constraints such as material availability and labor capacity. Data on production requirements for six furniture types—dining tables, chairs, sofa sets, dressing tables, shoe racks, and beds—were collected from the furniture shop's operational records.

The model formulation involved defining decision variables for each furniture type and constructing the objective function based on profit contributions. Constraints were introduced for resources like wood, fabric, paint, sun mica, stuffing, accessories, and labor hours. Non-negativity constraints ensured feasibility. The model was solved using Excel to derive the optimal production mix.

The research design is quantitative, focusing on numerical optimization. Results are interpreted to provide actionable insights for efficient resource utilization and profit maximization. The methodology ensures reliability through precise mathematical modeling and validation using real data.

Problem Assumption

Six main types of furniture to be produced are as follows:

- Dining tables, chairs, sofa sets, dressing tables, shoe racks and beds.
- There is limited availability of key materials like wood, fabric, paint, sun mica, stuffing, accessories, and labor.
- The production of each item requires specific amounts of these materials.
- The goal is to allocate resources efficiently to maximize profit while meeting all material and production constraints.

Table 1- Production requirements for six furniture types

	PRODUCTS						
	Dining Table (unit)	Chairs (unit)	Sofa Set (unit)	Dressing Table (unit)	Shoe Rack (unit)	Bed (unit)	Total Availability
Wood (cubic feet)	15	6	12	8	10	20	1200 cubic feet

Fabric (square feet)	0	8	40	25	15	20	1800 square feet
Paint (liters)	4	2	3	3	2	5	250 liters
Sun mica (sheets)	3	1	2	2	1	3	350 sheets
Stuffing (kilograms)	8	3	4	5	2	6	600 kilograms
Accessories (units)	5	2	3	4	3	2	400 units
Labor Cost per Unit (hours)	10	4	8	6	5	12	1500 hours
Profit per Unit (Rupees)	1500	600	2500	3000	2000	3500	

Model Formulation

Let:

x_1 = Number of dining tables produced

x_2 = Number of chairs produced

x_3 = Number of sofa sets produced

x_4 = Number of dressing tables produced

x_5 = Number of shoe racks produced

x_6 = Number of beds produced

Objective Function:

Maximize $\mathbf{Z = 1500x_1 + 600x_2 + 2500x_3 + 3000x_4 + 2000x_5 + 3500x_6}$

Subject to Constraints:

1. Wood Constraint:
$15x_1 + 6x_2 + 12x_3 + 8x_4 + 10x_5 + 20x_6 \leq 1200$

2. Fabric Constraint:
$0\,x_1 + 8x_2 + 40x_3 + 25x_4 + 15x_5 + 20x_6 \leq 1800$

3. Paint Constraint:
$4x_1 + 2x_2 + 3x_3 + 3x_4 + 2x_5 + 5x_6 \leq 250$

4. Sun mica Constraint:
$3x_1 + x_2 + 2x_3 + 2x_4 + x_5 + 3x_6 \leq 350$

5. Stuffing Constraint:
$8x_1 + 3x_2 + 4x_3 + 5x_4 + 2x_5 + 6x_6 \leq 600$

6. Accessories Constraint:
$5x_1 + 2x_2 + 3x_3 + 4x_4 + 3x_5 + 2x_6 \leq 400$

7. Labor Cost Constraint:
$10x_1 + 4x_2 + 8x_3 + 6x_4 + 5x_5 + 12x_6 \leq 1500$

8. Non-Negativity Constraints:

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Interpretation of Results

The optimal production mix derived from the model indicates the quantities of each product that should be produced to maximize profits while staying within material constraints. The results will guide the furniture production in resource allocation, ensuring profitability and efficient use of materials.

Research Findings

The above linear programming model was solved by EXCEL, which gives an optimal solution of: $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 3.70$, $x_5 = 107.40$, $x_6 = 4.81$ and maximum **Z = 2,42,777.78.** This solution indicates that prioritizing the production of shoe racks, dressing tables, and beds maximizes profitability while adhering to the given constraints.

Conclusion

The linear programming model, solved using Excel, demonstrates the effectiveness of optimization techniques in maximizing profitability in furniture production. The optimal solution highlights that focusing on the production of **shoe racks, dressing tables, and beds** yields the highest profit, with a maximum value of **₹2,42,777.78** while adhering to material and labour constraints. This study confirms that linear programming is a practical and powerful tool for resource allocation and decision-making in manufacturing. The results emphasize the importance of strategic production planning to achieve profitability and provide a framework for further optimization in similar industries.

Future Scope

Future research can explore dynamic models incorporating market demand fluctuations and seasonal variations. Expanding the scope to include cost analysis for material procurement and logistics can enhance the model's applicability. Automating LP implementation in production processes could further optimize resource use and improve efficiency. The methodology can also be adapted for other industries facing similar challenges.

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