**Mathematical Modelling of Poverty, Cybercrime, and Prostitution Dynamics in South-South, Nigeria**

**Abstract**

This study used a compartmentalized mathematical model to evaluate and establish the complex relationships existing between poverty, cybercrime, and prostitution. The model explains the dynamics of socioeconomic factors using the concepts of ordinary differential equations (ODE). The Routh Array criterion was adopted to perform stability analysis and to identify the stable state of the dynamical system. Numerical simulations were performed to illustrate the effects of various policy interventions, such as economic empowerment and law enforcement measures on the system. Findings showed that enhanced intervention strategies could result in significant poverty alleviation, reduction of cybercrime rate, and decrease in the number of those engaged in prostitution. This assessment demonstrates that poverty reduction strategies successfully limit the progression of these moral issues within the region. The model functions as an essential decision-making instrument that helps policymakers create efficient socioeconomic programmes.

*Keywords: Ordinary Differential Equation, Routh array criterion, Stability Analysis, Prostitution, Poverty, Intervention*

**Introduction**

Mathematical Modelling is loosely regarded as the implementation of mathematics in solving unstructured problems in real life. According to Dunder et al (2012), a model refers to the representation of reality or physical life with several meaningful symbols, while modelling is the process through which a model is developed and validated. The modelling process is preceded by a fairly complicated real-life situation, followed by problem analysis and mathematicalization, simulated solutions, validation, and implementation. Models have found wide applications in diverse fields of life. For instance, Aghanenu et al. (2022) simulated the effect of imperfect vaccines on the spread of COVID-19 in Nigeria, while Urumese and Igabari (2023) investigated the impact of community lockdown alongside social distancing on the spread of the pandemic. Furthermore, in Ohwojeheri et al (2024), a compartmentalized epidemiological model was adapted to explain the spread of social diseases such as poverty and crime, while the applications and challenges associated with linear models are discussed in Onyemarin et al. (2023) on the area of infant mortality, and Osemeke et al. (2024), on the core assumptions of such models. Such adaptations of mathematical and statistical models have been relatively efficient in explaining and providing strategies to minimize the undesirable effects of the associated vices.

Without any atom of doubt, poverty as a social disease, continues to thrive as an enduring and persistent challenge for the people living in less developed nations such as Nigeria. While the Nation is blessed with an abundance of natural resources, a great proportion of its citizens remain impoverished. Poverty has been defined as lacking the resources to meet basic needs, such as food, water, shelter, and healthcare. Widespread and deep-rooted poverty existing side by side with wealth can easily be explained by poor governance, maladaptive corruption, and dysfunctional access to education and other economically beneficial infrastructures. These patterns of economic decline have compelled a wide segment of the population, especially the youth, to turn towards desperate measures that include cybercrime and prostitution.

In Nigeria, cybercrime is popularly known as ‘Yahoo Yahoo’ and illustrates a major social problem, especially for the unemployed sector of the youth population, which sees it as a means to get out of poverty. The rise of cybercrime in the South-South region of Nigeria stems from improved Internet accessibility, non-existent regulations, and an intense cultural change that, in some cases, romanticizes fraud as a method to attain wealth. This crisis, combined with inflation, rampant underemployment, and an overall decrease in viable economic prospects, has exacerbated the circumstances that foster online fraud. Although various restrictions have been attempted by law enforcement agencies, the absence of viable sustainable economic options for the youth remains an enduring issue.

Always, prostitution has emerged as a strategy of survival for many poor people, especially young women. In the region, there is widespread prostitution due to poverty, domestic socio-economic burdens, gender discrimination, and limited educational and employment opportunities. Many prostitutes work under dangerous and exploitative circumstances that greatly endanger their health, open them up to abuse, and subject them to social stigma. While illegal measures are in place to curb prostitution, the fundamental factors, such as poverty and economic adversity, are generally overlooked, rendering such intervention ineffective in the long run.

One aspect cannot be solved independently from the others due to the intertwinement of prostitution, cybercrime, and poverty. Discipline-based approaches through traditional perspectives target cybercriminals and sex workers by employing the method of arrest yet fail to resolve the underlying socioeconomic causes of these actions. To develop an integrative appreciation of such conditions and their interconnectedness with one another, there is a need for a mathematical modelling strategy. A system of differential equations serves to investigate individual transitions among poverty, cybercrime, and prostitution states alongside studying policy intervention effects through poverty alleviation programs, employment promotion strategies, and enhanced law enforcement measures.

**Literature Review**

Current literature investigates how economic depletion creates a connexion between poverty, cybercrime, and prostitution (Nnatu, S. (2018); Idowu & Madaki (2021). According to the economic strain theory (Merton, 1938), financial difficulties make people choose illegal survival strategies which frequently lead them to engage in cyber fraud and prostitution. In alignment with Becker (1968) rational choice theory states that individuals commit offenses when they believe the advantages exceed the dangers they face. The research conducted by Adebayo and Adepoju (2022) as well as Alabi et al. (2023) reveals how unemployment and digital accessibility contribute to the spread of internet fraud in Nigerian society. Research shows that unemployed young people embark on cybercriminal conduct to maintain their existence. Nnatu, S. (2018) demonstrates that economic need drives individuals to engage in sex work primarily within areas facing extreme poverty and high rates of unemployment.

Several modern studies are concerned with essential cybersecurity and poverty eradication techniques using machine learning, blockchain, and IoT technology. For instance, Bhuiyan et al. (2024) reveal that the success of small and medium enterprises (SMEs) depends heavily on digital transformation, where different emerging technologies serve to boost their strategic approaches and outcomes. The systematic evaluation in Bhuiyan & Akter (2024) demonstrated how blockchain technology supports Smart Bangladesh development through multiple applications which create future opportunities for advancement, and also in the same 2024, Rahman et al. exposed both COVID-19 pandemic risks and possibilities for recovery which affected small and medium enterprises (SMEs) throughout Bangladesh.

Mathematical models have been used to analyze, predict, and suggest how poverty levels can be reduced to the barest minimum. In the quest to reduce the rate of poverty and drug addiction among the people living in Sylhet, Bangladesh, to the barest minimum, Sakib et al. (2017) developed a compartmental model to investigate the influence of governmental and non-governmental intervention programmes to address the stated challenges in this region. The population was classified into five groups in their model. The classes include the non-impoverished class, the poverty class, the drug-addicted class, the rehabilitation class, and the recovery class. Their research outcome revealed that the research objective, which is a reduction in the poverty and drug addiction rate, can be significantly reduced with the help of intervention programmes. Islam et al. (2017) further improved on Sakib et al. (2017) by including snatching and studying the influence of intervention programs on the rate of poverty, drug addiction, and snatching in the same region of Bangladesh. Their simulation result shows that intervention programmes, either from the government, individuals, or religious organizations, can reduce the rate of poverty, drug addiction, and snatching to the barest minimum.

Oduwole & Shehu (2013), in their work, developed a compartmental model that addresses both poverty and prostitution in Nigeria. The population was classified into five (5) compartments. The outcome of their research shows that prostitution and poverty can be subdued with the help of intervention programmes. Using the same compartmental classifications used by Oduwole & Shehu (2013) but with a different flow diagram, Akinpelu & Ojo (2016) findings also revealed government intervention is necessary in controlling the rate and spread of poverty.

Due to the strong relationship between poverty and crime in West Malaysia, Roslan et al. (2018) developed a mathematical model that considered three classes of people, namely, poverty, poor, and crime classes. Their simulation result shows that a high level of intervention from the government will reduce crime cases and poverty rates. We have also previously developed a model in Ohwojeheri et al. (2024) that described the interplay between poverty and cybercrime, where the study revealed that high interventions result in both poverty and cybercrime reduction, which is very important to our current study as we intend to advance the model by adding prostitution as a variable.

This study builds on these works by developing a more comprehensive mathematical model that accounts for socioeconomic transitions, policy interventions, and stability analysis as it aims to investigate the dynamics of poverty, cybercrime, and prostitution in South-South, Nigeria, and tries to obtain a useful approach to minimizing cybercrime and prostitution while reducing poverty in South-South, Nigeria.

**Model Development**

This model considered only seven classes of people in the South-South region of Nigeria. The classes considered includes; non-impoverished class (N(t)), poverty class (P(t)), prostitution class (U(t)), cybercrime class(C(t)), disease infected class(I(t)), jailed class (J(t)), and rehabilitation/recovery class (R(t)). The selected classes are dynamic i.e., they are meant to change with time. Based on this fact, sets of first-order ordinary differential equations were developed from the flow diagram (Fig 1) which are shown in equations 1 to 7. Apart from those seven model variables listed earlier, there are still some other model parameters in Fig 1. A detailed description of the model variables and parameters is given in Tables 1 and 2 respectively. The model was established on the following assumptions:

1. There will be movement from N(t) class to P(t) class at the rate of $β$ due to an increase in the rate of unemployment or underemployment. At times some government policies may lead to this movement also.
2. The tendency is very high that people will go into U(t) and C(t) classes as a result of poverty at the rate of $σ$ and $γ$ respectively.
3. There is the probability (though very low) that some individual in the N(t) class will also move to U(t) and C(t) classes at the rate of $ρ$ and $α$ respectively.
4. Some in the U(t) have a very high tendency to be infected with sexually transmitted diseases at the rate of $ϕ$.
5. There is probability that someone in the C(t) class will be caught and jailed at the rate of $φ$.
6. There is a probability that someone who has been rehabilitated can still go back to prostitution and cybercrime at the rate of $ε$ and $ω$ respectively due to the influence of their old friends that are still in those classes.



Fig. 1: Flow diagram for the system.

Table1: Description of model variables

|  |  |
| --- | --- |
| $Variables$  | $Description$  |
| $T$  | $Total population$  |
| $N(t)$  | $Non-impoverished class$  |
| $P(t)$  | $Poverty class$  |
| $U(t)$  | $Prostitution class$  |
| $C(t)$  | $Cyber crime class$  |
| $I(t)$  | $Disease infected class$  |
| $J(t)$  | $Jailed class$  |
| $R(t)$  | $Rehabilitation class$  |

Table 2: Description of model parameters

|  |  |
| --- | --- |
| Parameters | Description |
| $β$  | The rate of movement from non-impoverished class $\left(N(t)\right)$ to poverty class $\left(P(t)\right)$ $ $ |
| $ρ$  | The rate at which individuals in the non-impoverished class$\left(N(t)\right)$ move into prostitution$\left(U(t)\right)$ |
| $α$  | The rate at which individuals in the non-impoverished class$\left(N(t)\right)$ engage cyber-crime class$\left(C(t)\right)$ |
| $σ$  | The rate of movement from poverty class$\left(P(t)\right)$ into prostitution class $\left(U(t)\right)$ |
| $γ$  | The rate at which individuals in the poverty class$\left(P(t)\right)$ get involved in cyber crime$\left(C(t)\right)$ |
| $δ$  | The rate at which people in the cyber-crime class$\left(C(t)\right)$ moved to prostitution class$\left(U(t)\right)$ |
| $Ω$  | The rate of movement from prostitution class$\left(U(t)\right)$ to cyber-crime class$\left(C(t)\right)$ |
| $ϕ$  | The rate at which people in the prostitution class$\left(U(t)\right)$ get infected with STD and move to infected class $\left(I(t)\right)$ |
| $φ$  | The rate at which people in the cyber-crime class$\left(C(t)\right)$ were caught by law enforcement agencies and moved to the jailed class$\left(J(t)\right)$ |
| $θ$  | The rate at which individual in the infected class $\left(I(t)\right)$ recovered and moved to rehabilitation class $\left(R(t)\right)$ |
| $τ$  | The rate at which individual in the jailed class $\left(I(t)\right)$ moved to rehabilitation class $\left(R(t)\right)$ |
| $ε$  | Transition rate from rehabilitation class $\left(R(t)\right)$ to prostitution class$\left(U(t)\right)$ |
| $ω$  | Transition rate from rehabilitation class $\left(R(t)\right)$ to cyber-crime class$\left(C(t)\right)$ |
| $ψ $  | The rate of movement from poverty class$\left(P(t)\right)$ moved to recovery class $\left(R(t)\right)$ |
| $k\_{1}$  | Disease induced death rate due to infection in prostitution class $\left(U(t)\right)$ |
| $k\_{2}$  | Disease induced death rate due to infection in cyber-crime class $\left(C(t)\right)$ |
| $k\_{3}$  | Disease induced death rate due to infection in disease infected class $\left(I(t)\right)$ |
| $k\_{4}$  | Disease induced death rate due to infection in jailed class $\left(J(t)\right)$ |
| $μ$  | Birth/Death rate |

$\frac{dN}{dt}=μT-\frac{ρNU}{T}-\frac{αNC}{T}-\left(β+μ\right)N$ (1)

$\frac{dP}{dt}=βN-\frac{σPN}{T}-\frac{γPC}{T}-\left(ψ+μ\right)P$ (2)

$\frac{dU}{dt}=\frac{ρNU}{T}+\frac{σPU}{T}+\frac{δCU\left(ρ+σ+ε\right)}{T}-\left(ϕ+μ+k\_{1}\right)U-\frac{ΩUC\left(γ+α+ω\right)}{T}$ (3)

$\frac{dC}{dt}=\frac{γPC}{T}+\frac{αNC}{T}+\frac{ΩUC\left(γ+α+ω\right)}{T}+\frac{ωRC\left(α+γ+Ω\right)}{T}-\frac{δCU\left(ρ+σ+ε\right)}{T}-\left(φ+μ++k\_{2}\right)C$ (4)

$\frac{dI}{dt}=ϕU-\left(θ+μ+k\_{3}\right)I$ (5)

$\frac{dJ}{dt}=φC-\left(τ+μ+k\_{4}\right)J$ (6)

$\frac{dR}{dt}=θI+τJ-\frac{εRU\left(ρ+σ+δ\right)}{T}-\frac{ωRC\left(α+γ+Ω\right)}{T}+ ψP-μR$ (7)

$T=N+P+U+C+I+J+R$ (8)

**Stability Test At Steady State**

A steady state simply refers to a state where the model variables do not change with time i.e. $\frac{dA}{dt}$=0, *A=N, P, U, C, I, J, & R*. Subjecting Eqn. 1 to 7 to the definition of steady state, gives:

$0=μT-\frac{ρNU}{T}-\frac{αNC}{T}-\left(β+μ\right)N=f\_{1}$ (9)

$0=βN-\frac{σPN}{T}-\frac{γPC}{T}-\left(ψ+μ\right)P$ $=f\_{2}$ (10)

$0=\frac{ρNU}{T}+\frac{σPU}{T}+\frac{δCU\left(ρ+σ+ε\right)}{T}-\left(ϕ+μ+k\_{1}\right)U-\frac{ΩUC\left(γ+α+ω\right)}{T}$ $=f\_{3}$ (11)

$0=\frac{γPC}{T}+\frac{αNC}{T}+\frac{ΩUC\left(γ+α+ω\right)}{T}+\frac{ωRC\left(α+γ+Ω\right)}{T}-\frac{δCU\left(ρ+σ+ε\right)}{T}-\left(φ+μ++k\_{2}\right)C=f\_{4}$ (12)

$0=ϕU-\left(θ+μ+k\_{3}\right)I=f\_{5}$ (13)

$0=φC-\left(τ+μ+k\_{4}\right)J=f\_{6}$ (14)

$0=θI+τJ-\frac{εRU\left(ρ+σ+δ\right)}{T}-\frac{ωRC\left(α+γ+Ω\right)}{T}+ ψP-μR$ $=f\_{7}$ (15)

To determine if the system is stable under steady-state conditions, firstly Jacobian matrix has to be defined:

$$JM=\left[\begin{matrix}\begin{matrix}\frac{∂f\_{1}}{∂N}&\frac{∂f\_{1}}{∂P}&\frac{∂f\_{1}}{∂U}\\\frac{∂f\_{2}}{∂N}&\frac{∂f\_{2}}{∂P}&\frac{∂f\_{2}}{∂U}\\\frac{∂f\_{3}}{∂N}&\frac{∂f\_{3}}{∂P}&\frac{∂f\_{3}}{∂U}\end{matrix}&\begin{matrix}\frac{∂f\_{1}}{∂C}&\frac{∂f\_{1}}{∂I}&\begin{matrix}\frac{∂f\_{1}}{∂J}&\frac{∂f\_{1}}{∂R}\end{matrix}\\\frac{∂f\_{2}}{∂C}&\frac{∂f\_{2}}{∂I}&\begin{matrix}\frac{∂f\_{2}}{∂J}&\frac{∂f\_{2}}{∂R}\end{matrix}\\\frac{∂f\_{3}}{∂C}&\frac{∂f\_{3}}{∂I}&\begin{matrix}\frac{∂f\_{3}}{∂J}&\frac{∂f\_{3}}{∂R}\end{matrix}\end{matrix}\\\begin{matrix}\frac{∂f\_{4}}{∂N}&\frac{∂f\_{4}}{∂P}&\frac{∂f\_{4}}{∂U}\\\frac{∂f\_{5}}{∂N}&\frac{∂f\_{5}}{∂P}&\frac{∂f\_{5}}{∂U}\\\begin{matrix}\frac{∂f\_{6}}{∂N}\\\frac{∂f\_{7}}{∂N}\end{matrix}&\begin{matrix}\frac{∂f\_{6}}{∂P}\\\frac{∂f\_{7}}{∂P}\end{matrix}&\begin{matrix}\frac{∂f\_{6}}{∂U}\\\frac{∂f\_{7}}{∂U}\end{matrix}\end{matrix}&\begin{matrix}\frac{∂f\_{4}}{∂C}&\frac{∂f\_{4}}{∂I}&\begin{matrix}\frac{∂f\_{4}}{∂J}&\frac{∂f\_{4}}{∂R}\end{matrix}\\\frac{∂f\_{5}}{∂C}&\frac{∂f\_{5}}{∂I}&\begin{matrix}\frac{∂f\_{5}}{∂J}&\frac{∂f\_{5}}{∂R}\end{matrix}\\\begin{matrix}\frac{∂f\_{6}}{∂C}\\\frac{∂f\_{7}}{∂C}\end{matrix}&\begin{matrix}\frac{∂f\_{6}}{∂I}\\\frac{∂f\_{7}}{∂I}\end{matrix}&\begin{matrix}\begin{matrix}\frac{∂f\_{6}}{∂J}\\\frac{∂f\_{7}}{∂J}\end{matrix}&\begin{matrix}\frac{∂f\_{6}}{∂R}\\\frac{∂f\_{7}}{∂R}\end{matrix}\end{matrix}\end{matrix}\end{matrix}\right] (16)$$

Substituting equations 9 to 15 into equation 16 gives:

$\left(-\frac{ρU}{T}-\frac{αC}{T}-\left(β+μ\right)\right) 0 -\frac{ρN}{T} -\frac{αC}{T} 0 0$ 0

$\left(β-\frac{σP}{T}\right) \left(-\frac{σN}{T}-\frac{γC}{T}-\left(ψ+μ\right)\right) 0 -\frac{γP}{T} 0 0 0 $

$\frac{ρU }{T} \left(\frac{σU}{T}\right) \left(\frac{ρN}{T}+\frac{σP}{T}+\frac{δC\left(ρ+σ+ε\right)}{T}-\left(ϕ+μ+k\_{1}\right)-\frac{ΩC\left(γ+α+ω\right)}{T} \right) 0 0 0 0 $

$\frac{αC}{T} \left(\frac{γC}{T}\right) \left(\frac{ΩC\left(γ+α+ω\right)}{T}-\frac{δC\left(ρ+σ+ε\right)}{T}\right)$ $\left(\frac{γP}{T}+\frac{αN}{T}+\frac{ΩU\left(γ+α+ω\right)}{T}+\frac{ωR\left(α+γ+Ω\right)}{T}-\frac{δU\left(ρ+σ+ε\right)}{T}-\left(φ+μ++k\_{2}\right)\right)$ 0 0 0

$0 0 ϕ 0 -\left(θ+μ+k\_{3}\right) $0 0

0 0 0 $φ$ 0 $-\left(τ+μ+k\_{4}\right)$ 0

0 $ψ$ $-\frac{εR\left(ρ+σ+δ\right)}{T}$ $\left(-\frac{ωR\left(α+γ+Ω\right)}{T}\right)$ $θ$ $τ$ $\left(-\frac{εU\left(ρ+σ+δ\right)}{T}-\frac{ωC\left(α+γ+Ω\right)}{T}\right)$

Secondly, the characteristic equation has to be defined. Following (Islam et al., 2017) approach the characteristic equation is given as:

$\left(s-\left(-\frac{ρU}{T}-\frac{αC}{T}-\left(β+μ\right)\right)\right)\left(s-\left(-\frac{σN}{T}-\frac{γC}{T}-\left(ψ+μ\right)\right)\right)\left(s-\left(\frac{ρN}{T}+\frac{σP}{T}+\frac{δC\left(ρ+σ+ε\right)}{T}-\left(ϕ+μ+k\_{1}\right)-\frac{ΩC\left(γ+α+ω\right)}{T} \right)\right)\left(s-\left(\frac{γP}{T}+\frac{αN}{T}+\frac{ΩU\left(γ+α+ω\right)}{T}+\frac{ωR\left(α+γ+Ω\right)}{T}-\frac{δU\left(ρ+σ+ε\right)}{T}-\left(φ+μ++k\_{2}\right)\right)\right)\left(s+\left(τ+μ+k\_{4}\right)\right)\left(s-\left(-\frac{εU\left(ρ+σ+δ\right)}{T}-\frac{ωC\left(α+γ+Ω\right)}{T}\right)\right)=0$ (18)

Let $\left(-\frac{ρU}{T}-\frac{αC}{T}-\left(β+μ\right)\right)=a$, $\left(-\frac{σN}{T}-\frac{γC}{T}-\left(ψ+μ\right)\right)=b$, $\left(\frac{ρN}{T}+\frac{σP}{T}+\frac{δC\left(ρ+σ+ε\right)}{T}-\left(ϕ+μ+k\_{1}\right)-\frac{ΩC\left(γ+α+ω\right)}{T} \right)=c$, $\left(\frac{γP}{T}+\frac{αN}{T}+\frac{ΩU\left(γ+α+ω\right)}{T}+\frac{ωR\left(α+γ+Ω\right)}{T}-\frac{δU\left(ρ+σ+ε\right)}{T}-\left(φ+μ++k\_{2}\right)\right)=d$, $\left(τ+μ+k\_{4}\right)=e$, $\left(-\frac{εU\left(ρ+σ+δ\right)}{T}-\frac{ωC\left(α+γ+Ω\right)}{T}\right)=f$

Equation (18) becomes

$\left(s-a\right)\left(s-b\right)\left(s-c\right)\left(s-d\right)\left(s-e\right)\left(s-f\right)=0$ (19)

Expansion of equation 19 gives:

$s^{6}+\left(-e-d-c-b-a-f\right)s^{5}+\left(bd+bc+ad+ac+ab+de+ce+be+ae+df+bf+ef+cf+af+cd\right)s^{4}+\left(-bcd-acd-abd-abc-cde-bde-bce-ade-ace-abe-cdf-bdf-bcf-adf-acf-abf-def-cef-bef-aef\right)s^{3}+\left(bcde+acde+abde+abce+abcd+bcdf+acdf+abdf+abcf+cdef+bdef+bcef+adef+acef+abef\right)s^{2}+\left(-bcdef-acdef-abdef-abcef-abcdf-abcde\right)s+\left(abcdef\right)=0$ (20)

Let $-e-d-c-b-a-f=a\_{1}$, $bd+bc+ad+ac+ab+de+ce+be+ae+df+bf+ef+cf+af+cd=a\_{2}$, $-bcd-acd-abd-abc-cde-bde-bce-ade-ace-abe-cdf-bdf-bcf-adf-acf-abf-def-cef-bef-aef=a\_{3}$, $bcde+acde+abde+abce+abcd+bcdf+acdf+abdf+abcf+cdef+bdef+bcef+adef+acef+abef=a\_{4}$, $-bcdef-acdef-abdef-abcef-abcdf-abcde=a\_{5}$, $abcdef=a\_{6}$

The characteristic equation becomes:

$s^{6}+a\_{1}s^{5}+a\_{2}s^{4}+a\_{3}s^{3}+a\_{4}s^{2}+a\_{5}s+a\_{6}=0$ (21)

Lastly, apply Routh array criterion to obtain condition for stability in equation (21)

$s^{6} 1 a\_{2 }a\_{4} a\_{6}$

$s^{5}$ $a\_{1}$ $a\_{3}$ $a\_{5}$ 0

$s^{4}$ $b\_{1}$ $b\_{2}$ $b\_{3}$ 0

$s^{3}$ $b\_{4}$ $b\_{5}$ $0$ 0

$s^{2}$ $b\_{6}$ $b\_{7}$ $0$ 0

$s^{1}$ $b\_{8}$ $0$ $0$ 0

$s^{0}$ $b\_{9}$ $0$ $ 0$ 0

$b\_{1}=-\frac{1}{a\_{1}}\left|\begin{matrix}1&a\_{2}\\a\_{1}&a\_{3}\end{matrix}\right|$ , $b\_{2}=-\frac{1}{a\_{1}}\left|\begin{matrix}1&a\_{4}\\a\_{1}&a\_{5}\end{matrix}\right|$, $b\_{3}=-\frac{1}{a\_{1}}\left|\begin{matrix}1&a\_{6}\\a\_{1}&0\end{matrix}\right|$, $b\_{4}=-\frac{1}{b\_{1}}\left|\begin{matrix}a\_{1}&a\_{3}\\b\_{1}&b\_{2}\end{matrix}\right|$, $b\_{5}=-\frac{1}{b\_{1}}\left|\begin{matrix}a\_{1}&a\_{5}\\b\_{1}&b\_{3}\end{matrix}\right|$, $b\_{6}=-\frac{1}{b\_{4}}\left|\begin{matrix}b\_{1}&b\_{2}\\b\_{4}&b\_{5}\end{matrix}\right|$, $b\_{7}=-\frac{1}{b\_{4}}\left|\begin{matrix}b\_{1}&b\_{3}\\b\_{4}&0\end{matrix}\right|$, $b\_{8}=-\frac{1}{b\_{6}}\left|\begin{matrix}b\_{4}&b\_{7}\\b\_{6}&0\end{matrix}\right|$, $b\_{9}=-\frac{1}{b\_{6}}\left|\begin{matrix}b\_{6}&b\_{7}\\b\_{8}&0\end{matrix}\right|$

The system will be stable if $b\_{i}\left(i=1,2,3,4,5,6,7,8,9\right)$ are non-negative while the system will be unstable if any of the values of $b\_{i}$ is negative.

**Numerical Simulation**

Here, the set of ordinary differential equations given in equations 1 to 7 was solved numerically using the 4th-order Runge-Kutta method in MATLAB environment. The baseline variables were obtained from both the Nigeria Bureau of Statistics and field survey for research purposes while parameter values were assumed after careful comparison with what is obtainable in literature as shown in the tables below:

Table 3: Initial values of the model variables

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variables | $$T$$ | $$N(0)$$ | $$P(0)$$ | $$U(0)$$ | $C(0)$  | $I(0)$  | $J(0)$  | $$R(0)$$ |
| Values in millions | 33.98 | 25.48 | 8.5 | 827700 | 260780 | 244709 | 83084 | 151660 |

Source: NBS (2019), Field Survey.

Table 4: Values of the model parameters

|  |  |
| --- | --- |
| Parameters | Values |
| $β$  | 0.078 |
| $ρ$  | 0.45 |
| $α$  | 0.31 |
| $σ$  | 0.29 |
| $γ$  | 0.32 |
| $δ$  | 0.41 |
| $Ω$  | 0.60 |
| $ϕ$  | 0.23 |
| $φ$  | 0.22 |
| $θ$  | 0.59 |
| $τ$  | 0.35 |
| $ε$  | 0.40 |
| $ω$  | 0.44 |
| $ψ $  | 0.09 |
| $k\_{1}$  |  0.03 |
| $k\_{2}$  | 0.02 |
| $k\_{3}$  | 0.02 |
| $k\_{4}$  | 0.0071 |
| $μ$  | 0.01392 |

Source: Assumed

Fig. 2: Dynamics of the different population classes under non-intervention programme

Fig. 3: Dynamics of the prostitution class under non-intervention programme

Figure 4: Dynamics of the rehabilitation class under non-intervention programme



Fig. 5: Dynamics of the jailed class under non-intervention programme



Fig. 6: Dynamics of the different population classes under an intervention programme



Fig. 7: Dynamics of the disease-infected class under an intervention programme

 Fig. 8: Dynamics of the prostitution class under an intervention programme



Fig. 9: Dynamics of the non-impoverished class under high and low intervention programme



Fig. 10: Dynamics of poverty class under high and low intervention programme



Fig. 11: Dynamics of the cybercrime class under high and low intervention programme



Fig. 12: Dynamics of the prostitution class under high and low intervention programme



Fig. 13: Dynamics of the disease-infected class under high and low intervention programme



Fig. 14: Dynamics of the jailed class under high and low intervention programme



Fig. 15: Dynamics of the rehabilitation class under high and low intervention programme

**Results and Discussion**

The following parameters, $ψ$, $θ$, and $φ$ were considered to be intervention parameters in Fig. 1. Firstly, the different populations in the figure were analysed without an intervention program by both governmental and non-governmental organizations i.e., $ψ=0$, $θ=0$, and $φ=0$. The result of the non-intervention program is graphically shown in Fig. 2. The dynamic of U, J, and R classes are not clearly shown in the figure therefore they were plotted separately in Fig. 3, 4, and 5 respectively for a better view. The following observations were drawn from Figs. 2 to 5:

* The population in the N class became stable after it had significantly reduced for a short period.
* The population in the P class significantly rises for a short period. After that, it reduces and becomes stable.
* The population in the C class rises for a certain period. After which it became stable.
* The population in the I class rises to a particular point and after which it begins to decline progressively and becomes stable.
* The population in the J class reduces sharply from the starting point for a short period. After that it became stable.
* There was an increase in the population of the U class. After which it reduces and becomes stable.
* After a short period from the initial point the population in the R class increased to a peak after which it reduced and became stable.

The simulation results under an intervention programme are summarised in Figs. 6 to 8. Under an intervention programme $ψ=0.09$, $θ=0.59$, and $φ=0.22$. Comparing the N class under intervention (Fig. 6) with those under non-intervention programme (Fig. 3) it was observed that the presence of an intervention programme increased the population in the non-impoverished class. When Figs. 6 to 8 were compared with Figs. 2 to 5 it was revealed that the presence of an intervention programme reduces the population of P, C, I, and U classes. There was a rise in the population of J and R classes under an intervention programme. A rise in the J class simply signifies that the arrest of people who are involved in criminal activities is intensified by law enforcement agencies while the rise in the R class signifies that more people were recruited into the different rehabilitation programmes.

Further study was carried out by simulating each population class under low and high intervention. Under low intervention $ψ=0.05$, $θ=0.31$, and $φ=0.10$ While under high intervention $ψ=0.11$, $θ=0.72$, and $φ=0.53$. Simulation results are presented in Fig. 9 to 15. It can be deduced from Fig. 9 that a high intervention programme will slightly increase the population of people a little bit above low intervention after a while. Figure 10 revealed that the population of people in the poverty class will significantly reduce after a while under high intervention. This simply implies that both governmental and non-governmental organizations should not relent in their effort to reduce the poverty rate in the South-South region of Nigeria. Figs. 11, 12, and 13 revealed that the presence of high intervention programmes can salvage the region from cybercrime, prostitution and those in disease-infected compartments. Fig. 14 shows that the population of people arrested by law enforcement agencies increased under the high-intervention programme while Fig. 15 revealed that more people will be recruited into the rehabilitation class under high-intervention programmes.

**Conclusion**

The research developed a compartmental mathematical model to analyse the mutual relationships between poverty, cybercrimes, and prostitution in the South-South region of Nigeria. The stability analysis of the system was done using the Routh array criterion. The seven (7) sets of ordinary differential equations derived from the system flow diagram were solved numerically using the 4th-order Runge-Kutta method under intervention and non-intervention programmes. The research findings revealed that intervention programmes have a positive impact on the classes. By economic empowerment or enhanced intervention programmes, we mean policies and initiatives targeted at raising people out of poverty. Education, skill acquisition, and job placement should deliberately form part of these programmes so that the practitioners reduce their economic incentives to engage in both cybercrime and prostitution. This assessment demonstrates that poverty reduction strategies successfully limit the progression of these moral issues within the region. The model functions as an essential decision-making instrument that helps policymakers create efficient socioeconomic programmes.

**Recommendation**

Both government authorities and law enforcement units should implement economic development programmes to fight poverty and provide societal development alternatives for sustainable livelihoods.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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